$$y''-2y'+5y=0$$

$$r^2-2r+5=0 \iff r=\frac{2\pm\sqrt{4-20}}{2} = 1\pm 2i$$

$$\boxed{y = e^x\left(C_1 \cos 2x + C_2 \sin 2x\right)}$$ ← Sol. gén. de l'éq. homogène associée.

Il nous manque une sol. part. de $y''-2y'+5y=25x$

D'après le tableau de décision : $\underset{\text{candidat}}{p(x)} = \underbrace{kx+m}_{\text{degré } 1} = y$ $\quad k, m \in \mathbb{R}$

$$(kx+m)''-2(kx+m)'+5(kx+m)=25x$$

$$0 \quad -2k+5kx+5m = 25x$$

$$\iff 5kx \underbrace{-2k+5m} = 25x+0$$

$$5k\,x + (-2k+5m) = 25\,x + 0$$

$5k=25 \qquad k=5$

$-2k+5m=0 \qquad 5m=10 \,/\, m=2$

$\Rightarrow$ $p(x) = 5x+2$ est sol. part. de $y''-2y'+5y = 25x$

$\Rightarrow$ La solution cherchée est :

$$\boxed{y = e^{x}\left(c_1 \cdot \cos 2x + c_2 \sin 2x\right) + 5x+2}$$

2.3 c

$$y' + \tan x \cdot y = \frac{1}{\cos x}$$

$$y' + \tan x \, y = 0 \Leftrightarrow y' = -\tan x \cdot y \Leftrightarrow \frac{y'}{y} = -\tan x$$

$$\Leftrightarrow y = k \cdot e^{-\int \tan x \, dx} = k \cdot e^{\ln(\cos x)} = k \cdot \cos x$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\ln(\cos x)$$

$\Rightarrow$ Sol générale de l'éq. hom. assoc. : $k \cdot \cos x$

Trouver une solution particulière de $y' + \tan x \cdot y = \frac{1}{\cos x}$

$$y' + a(x)y = b(x)$$

$\boxed{\text{Facteur intégrant: } e^{\int a(x)\,dx}}$

$$\underbrace{e^{\int a(x)\,dx} \cdot y' + a(x) \cdot e^{\int a(x)\,dx} \cdot y}_{} = b(x) \cdot e^{\int a(x)\,dx}$$

$$\left(e^{\int a(x)\,dx} \cdot y\right)' = b(x) \cdot e^{\int a(x)\,dx}$$

$$e^{\int a(x)\,dx} = e^{\int \tan(x)\,dx} = e^{-\ln(\cos x)} = \frac{1}{\cos x}$$

$$\Rightarrow \left(\frac{1}{\cos x} y\right)' = \frac{1}{\cos x} \cdot \frac{1}{\cos x}$$

$$\Leftrightarrow \frac{1}{\cos x} y = \int \frac{1}{\cos^2 x}\,dx \Leftrightarrow y = \cos x \int \frac{1}{\cos^2 x}\,dx$$

$$= \cos x \cdot \tan x$$

$$= \sin x$$

Solution cherchée: $\boxed{\sin x + k\cos x}$

$$y' + a(x)\,y = b(x)$$

$\boxed{2.3\,b}$

$a(x) = -\frac{1}{x} \quad b(x) = \frac{x-1}{x} e^{x}$

$x \neq 0$

$$\boxed{x y' - y = (x-1) e^{x}} \iff y' - \frac{1}{x} y = \frac{x-1}{x} e^{x}$$

Ég. hom.

$$y' - \frac{1}{x} y = 0 \iff \ln|y| = \int -a(x)\,dx = \int \frac{1}{x}\,dx = \ln x + C$$

$$\iff y = k \cdot x$$

Sol. générale de l'ég. hom. assoc.

$y'' + y' - 2y = 2x$

$y(0) = 0$

$y'(0) = 1$

$y = c_1 e^{x} + c_2 e^{-2x} - x - \frac{1}{2}$

$y(0) = 0 \Leftrightarrow c_1 + c_2 - \frac{1}{2} = 0$

$y' = c_1 e^{x} - 2c_2 e^{-2x} - 1$

$y'(0) = 1 \Leftrightarrow c_1 - 2c_2 - 1 = 1$

$\begin{cases} c_1 + c_2 = \frac{1}{2} \\ c_1 - 2c_2 = 2 \end{cases}$

$3c_2 = \frac{1}{2} - 2 = -\frac{3}{2}$

$\boxed{c_2 = -\frac{1}{2} \quad c_1 = 1}$

$\Rightarrow y = e^{x} - \frac{1}{2} e^{-2x} - x - \frac{1}{2}$

$$y' + a(x)\,y = b(x)$$

$$y' + a(x)\,y = 0 \iff y' = -a(x)\,y \iff \frac{y'}{y} = -a(x)$$

$$\ln|y| = \int -a(x)\,dx \iff y = k \cdot e^{\int -a(x)\,dx}$$

$$x\,dx + y e^{-x}\,dy = 0$$

$$-y e^{-x}\,dy = x\,dx$$

$$-y\,dy = e^{x} \cdot x\,dx$$

$1e^{x} + (x-1)e^{x} = xe^{x}$ ✓

$$-\frac{1}{2}y^2 = \int x e^{x}\,dx = (x-1)e^{x} + c$$

$$y^2 = -2(x-1)e^{x} + c'$$

$$y = \pm\sqrt{-2(x-1)\cdot e^{x} + c'}$$

$$y(0)=1 \quad \Rightarrow \quad y=\sqrt{-2(x-1)e^{x}+c'}$$

$$y(0)=\sqrt{-2(0-1)e^{0}+c'} = \sqrt{2+c'} = 1$$