TE algèbre linéaire : 12/02/2024

6.1
6.2
6.3 Dossier supplémentaire
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6.6
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1.4.1
1.4.2
(1.4.3)
1.4.8 abefh
1.4.11

Déterminants et matrice inverse

$$M = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} \qquad \det(M) = 1\cdot 1 - (3)\cdot(-2)$$

$$\underset{\mathbb{R}}{\Uparrow} \qquad = 1+6 = 7$$

$$\mathbb{R}^2 \xrightarrow{m} \mathbb{R}^2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto M \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ 3x_1 + x_2 \end{pmatrix}$$

$$m(x_1 ; x_2) = (x_1 - 2x_2 ; 3x_1 + x_2)$$

$$\boxed{M \text{ inversible} \Leftrightarrow \det(M) \neq 0}$$

$$? \exists A \in M_2(\mathbb{R}) \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

I_2 unité de $M_2(\mathbb{R})$ pour la multip.

tq. $$\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & 3 \end{pmatrix}$$

$$\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$a_{22}a_{33} - a_{32}a_{23}$$

$$= a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - \left(a_{21}a_{12}a_{33} - a_{21}a_{32}a_{13}\right) + \left(a_{31}a_{12}a_{23} - a_{31}a_{22}a_{13}\right)$$

1.4.1

1.4.2

$$\begin{vmatrix}1&2&1\\0&1&1\\0&2&-3\end{vmatrix} = 1\cdot\begin{vmatrix}1&1\\2&-3\end{vmatrix} - 0\cdot\cancel{\begin{vmatrix} \; \end{vmatrix} + 0\begin{vmatrix} \; \end{vmatrix}}$$

$$\det\begin{pmatrix}1&2&1\\2&1&1\\3&-1&2\end{pmatrix} =$$

$$\begin{vmatrix}1&2&1\\2&1&1\\3&-1&2\end{vmatrix} = \begin{vmatrix}1&2&1\\0&-3&-1\\0&-7&-1\end{vmatrix} = \begin{vmatrix}-3&-1\\-7&-1\end{vmatrix}$$

$$\boxed{\begin{matrix}L_2 \leftarrow L_2 - 2L_1\\ L_3 \leftarrow L_3 - 3L_1\end{matrix}}$$

$$= 3 - 7 = -4$$

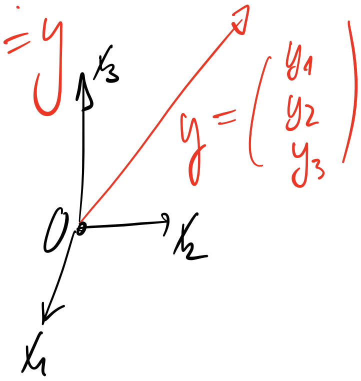
$$-6 = \begin{vmatrix}1&2\\2&-2\end{vmatrix} = 2\cdot\begin{vmatrix}1&2\\1&-1\end{vmatrix} = -3\cdot 2$$

Image de l'application donnée par

$$H=\begin{pmatrix}1&-2&1\\2&1&1\\1&-1&1\end{pmatrix}$$

$$H^{-1}\quad\left(\begin{array}{ccc|ccc}1&-2&1&1&0&0\\2&1&1&0&1&0\\1&-1&1&0&0&1\end{array}\right)$$

$$\exists x \quad H\cdot x \overset{?}{=} y$$



$$\mathbb{R}^3 \xrightarrow{H} \mathbb{R}^3$$

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \longmapsto H\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}$$

$$\mathrm{Im}(H)=\begin{cases}\mathbb{R}^3\\ \text{plan par } (0;0;0)\\ \text{droite par } (0;0;0)\\ (0;0;0)\end{cases}$$

$$\begin{pmatrix}1\\2\\1\end{pmatrix},\begin{pmatrix}-2\\1\\-1\end{pmatrix},\begin{pmatrix}1\\1\\1\end{pmatrix} \quad \text{lin. indép.}$$

$$H=\begin{pmatrix}1&-2&1\\2&1&1\\1&-1&1\end{pmatrix}\sim\begin{pmatrix}1&-2&1\\0&5&-1\\0&1&0\end{pmatrix}\sim\begin{pmatrix}1&-2&1\\0&1&0\\0&0&-6\end{pmatrix}\Rightarrow \mathrm{Im}(H)=\mathbb{R}^3$$

$$H\cdot x = H\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1&-2&1\\2&1&1\\1&-1&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=x_1\begin{pmatrix}1\\2\\1\end{pmatrix}+x_2\begin{pmatrix}-2\\1\\-1\end{pmatrix}+x_3\begin{pmatrix}1\\1\\1\end{pmatrix}$$

Matrice inverse

$$M = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$$

$\det(M) = 7 \neq 0 \Rightarrow M$ est inversible

$\Rightarrow \exists\, M^{-1} \in M_2(\mathbb{R})$ tq. $\boxed{M \cdot M^{-1} = M^{-1} \cdot M =} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Algorithme pour trouver M^{-1}

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) \qquad L_2 \leftarrow L_2 - 3L_1$$

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 7 & -3 & 1 \end{array}\right) \qquad L_2 \leftarrow \frac{L_2}{7}$$

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -3/7 & 1/7 \end{array}\right) \qquad L_1 \leftarrow L_1 + 2L_2$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 1/7 & 2/7 \\ 0 & 1 & -3/7 & 1/7 \end{array}\right)$$

M^{-1}

$$\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1/7 & 2/7 \\ -3/7 & 1/7 \end{pmatrix} \checkmark = \begin{pmatrix} 1/7 + 6/7 & 2/7 - 2/7 \\ 3/7 - 3/7 & 6/7 + 1/7 \end{pmatrix}$$

$$\boxed{M^{-1} = \frac{1}{7}\begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix}} \qquad = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$