$\boxed{\text{Sous-espaces de solutions}}$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$A \cdot X = 0$

- $X = 0$
- Ss-espace

$$\begin{aligned} x_1 &= x_2 - 2x_3 - 1 \\ x_2 &= x_3 + 1 \\ x_3 &= 2 \end{aligned} \qquad \begin{aligned} x_1 &= -2 \\ x_2 &= 3 \\ x_3 &= 2 \end{aligned}$$

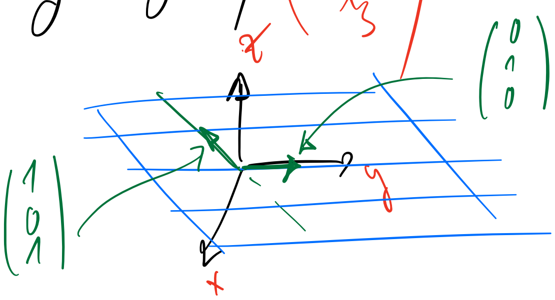
$$A \cdot X = 0 \iff \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$A \cdot X = 0 \iff \boxed{x_1 - x_3 = 0}$

plan $\boxed{x - z = 0}$

$ax + by + cz + d = 0$

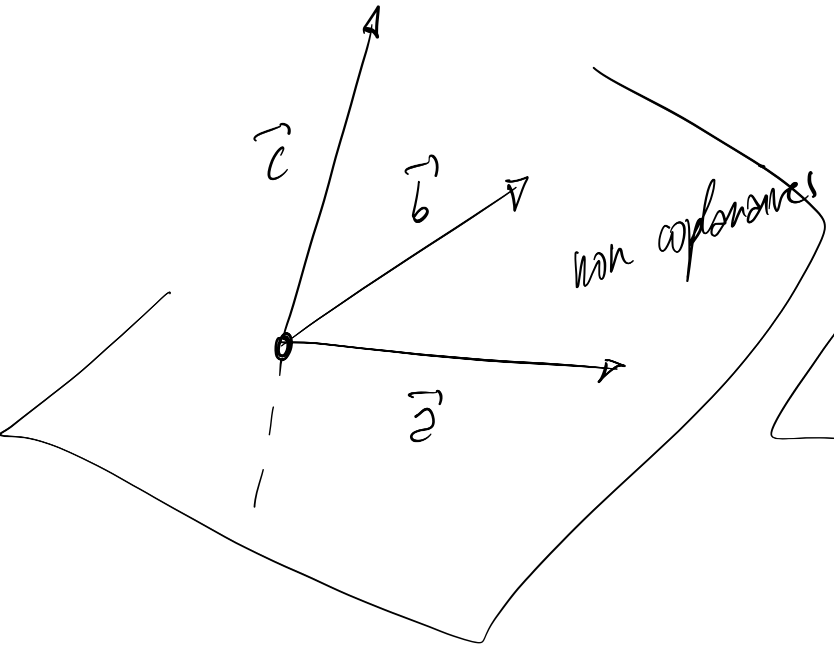


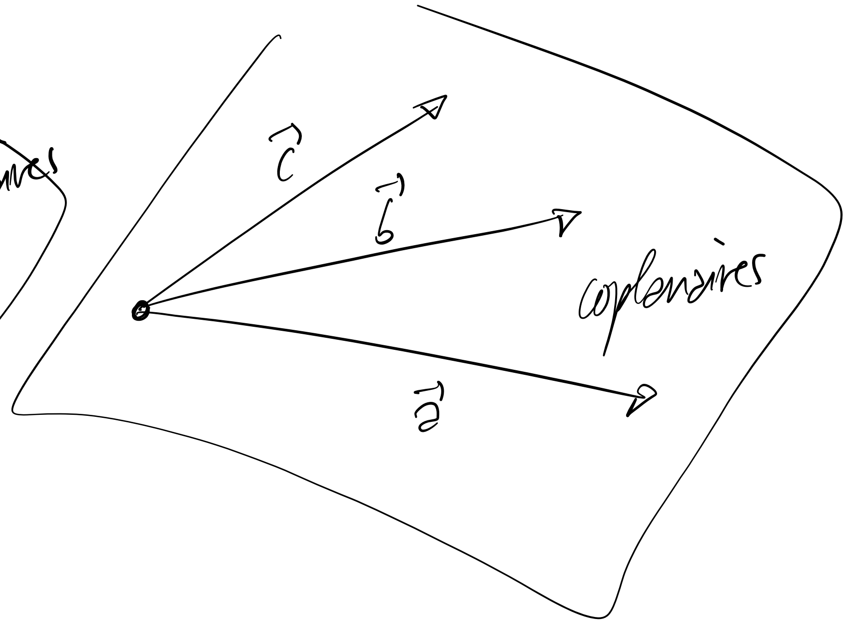
$$\begin{cases} x_1 = x_3 = k\cdot 1 + l\cdot 0 \\ x_2 = x_2 = k\cdot 0 + l\cdot 1 \\ x_3 = x_3 = k\cdot 1 + l\cdot 0 \end{cases} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = k\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + l\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Sous-espace de sols : plan engendré par $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\boxed{x_1 - x_3 = 0}$$ éq. cartésienne

$$\begin{aligned} x_1 &= 0\cdot x_2 + 1\, x_3 \\ x_2 &= 1\cdot x_2 + 0\cdot x_3 \\ x_3 &= 0\cdot x_2 + 1\cdot x_3 \end{aligned}$$ éq. paramétriques





INDÉPENDANCE LINÉAIRE

$\vec{u}_1, \dots, \vec{u}_k$ sont linéairement indépendants ssi

$$\sum_{i=1}^{k} x_i \cdot \vec{u}_i = \vec{0} \implies x_1 = 0, \dots, x_k = 0$$

$$x_i \in \mathbb{R}$$

<u>Exemple:</u> $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ lin. indép.?

$$x\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + z\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Si le seul sol. est $(0;0;0)$, les vect. sont lin. indép.

$\boxed{\text{À étudier :}}$

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & -1 \\ 3 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & -2 & 4 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{pmatrix} = A$$

$$\Rightarrow A \cdot X = 0 \Leftrightarrow X = (0;0;0)$$

$$\Rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \text{ sont lin. indép.}$$

<u>Cas particulier :</u> $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ $c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

sont lin. indép.

$$\Leftrightarrow \det\left(\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \right) \neq 0$$

<u>Exemple :</u>

$$\det\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & -1 \\ 3 & 1 & 1 \end{pmatrix} = 1 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} - 2 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= 2 - 4 + 3 = 1 \neq 0$$