

Intégrales

- MS {
- ① Tables
 - ② Substitution « dérivée interne » $f(g(x)) g'(x)$
- MR {
- ③ Changement de variable (Tables)
 - ④ Par parties
 - ⑤ Éléments simples $\left(\int \frac{P(x)}{Q(x)} dx, P, Q \text{ des polynômes} \right)$

Aires et volumes $\triangle$ formule autour de G par $\heartsuit$

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$$\int \frac{x^2 - 2x + 3}{x^2 - 1} dx$$

$$\frac{P(x)}{Q(x)}$$

$$P(x) \mid Q(x)$$

$$\begin{array}{r|l} x^2 - 2x + 3 & x^2 - 1 \\ \hline x^2 & -1 \\ \hline & -2x + 4 \end{array}$$

$$x^2 - 2x + 3 = (x^2 - 1) \cdot 1 - 2x + 4 \quad \cdot \frac{1}{x^2 - 1}$$

$$\frac{x^2 - 2x + 3}{x^2 - 1} = 1 + \boxed{\frac{-2x + 4}{x^2 - 1}}$$

$$\int \frac{-2x + 4}{x^2 - 1} dx = - \int \frac{2x}{x^2 - 1} dx + \boxed{\int \frac{4}{x^2 - 1} dx}$$
$$- \ln |x^2 - 1|$$

$$\frac{4}{x^2 - 1} = \frac{4}{(x+1)(x-1)} = \frac{2}{x+1} + \frac{6}{x-1}$$

$$\frac{4 \cdot \cancel{(x+1)}}{\cancel{(x+1)}(x-1)} = 2 + \frac{6(x+1)}{x-1}$$

$$\boxed{x = -1} \quad \frac{4}{-2} = 2 \quad 2 = -2$$

$$\frac{4 \cdot \cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{2(x-1)}{(x+1)} + 6$$

$$\boxed{x = 1} \quad \frac{4}{2} = 6 \quad 6 = 2$$

$$\frac{4}{(x+1)(x-1)} = \frac{-2}{x+1} + \frac{2}{x-1}$$

$$\Rightarrow \int \frac{x^2 - 2x + 3}{x^2 - 1} dx = x - \ln|x^2 - 1| - 2 \ln|x+1| + 2 \ln|x-1| + C$$

$$\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{a}{x+1} + \frac{bx+c}{x^2-x+1}$$

$$\frac{\frac{1}{x^3} \cdot (x^2 - 2x + 3)}{\frac{1}{x^3} \cdot (x^3 + 1)} = \frac{\frac{x^2}{x^3} - \frac{2x}{x^3} + \frac{3}{x^3}}{\frac{x^3}{x^3} + \frac{1}{x^3}} = \frac{\frac{1}{x} - \frac{2}{x^2} + \frac{3}{x^3}}{1 + \frac{1}{x^3}}$$

$$\begin{array}{l} \downarrow \text{B.-H.} \\ \frac{2x-2}{3x^2} \xrightarrow{\text{B.-H.}} \frac{2}{6x} \xrightarrow{x \rightarrow \infty} 0 \end{array}$$

$$\begin{array}{l} \downarrow x \rightarrow \infty \\ \frac{0 - 0 + 0}{1 + 0} = 0 \end{array}$$