$$\int (2x+b)^k \, dx = \frac{1}{2}\int (2x+b)^k \cdot 2 \cdot dx$$

$$= \frac{1}{2} \cdot \frac{1}{k+1}(2x+b)^{k+1} + C$$

$$\int f(g(x)) \cdot g'(x) \, dx = F(x) + C$$

si $F' = f$

$$\int \boxed{x \cdot \sin x}\, dx \stackrel{?}{=}$$

PAR PARTIES

$$\int \boxed{x \cdot \sin x}\, dx$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$x\,(-\cos x) \qquad 1 \cdot (-\cos x) \qquad \boxed{x \cdot \sin x}$$

$$f \cdot g = \int f' g + \int f \cdot g'$$

$$C + (-x \cdot \cos x) = \int (-\cos x)\, dx + \int x \sin x\, dx$$

$$C + (-x \cos x) = -\sin x + \int x \sin x\, dx$$

$$\Rightarrow \int x \sin x\, dx = \sin x - x \cos x + C$$

$$\int \cos^2 x \, dx = \int \cos x \cdot \cos x \, dx$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$(\sin x \cos x)' = (-\sin x) \sin x + \cos x \cdot \cos x$$

$$\sin x \cos x = -\int \sin^2 x \, dx + \int \cos^2 x \, dx$$

$$\cos^2 x + \sin^2 x = 1 \iff \sin^2 x = 1 - \cos^2 x$$

$$C + \sin x \cos x = -\int (1 - \cos^2 x) \, dx + \int \cos^2 x \, dx$$

$$C + \sin x \cos x = -x + \int \cos^2 x \, dx + \int \cos^2 x \, dx$$

$$2 \int \cos^2 x \, dx = x + \sin x \cos x + C$$

$$\boxed{\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{2} \sin x \cos x + C}$$

$$= \frac{1}{2} x + \frac{1}{2} \cdot \frac{1}{2} \sin 2x + C$$

$$= \frac{1}{2} x + \frac{1}{4} \sin 2x + C$$

$$\int \sin^4 x \, dx = \int \sin^2 x \cdot \sin^2 x \, dx$$

$$(f \cdot g)' = f'g + fg'$$

$$2 \sin x \cos x \cdot \int \sin^2 x + \sin^2 x \cdot \sin^2 x$$

$$\Big(\underbrace{\sin^2 x \cdot \tfrac{1}{2}(x - \sin x \cos x)}_{h(x)}\Big)' = 2 \sin x \cos x \, \tfrac{1}{2}\left(x - \sin x \cos x\right) + \sin^4 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$h(x) = x \sin x \cos x - \sin^2 x \cos^2 x + \sin^4 x$$

$$\tfrac{1}{2} x \sin 2x + \left(\sin^2 x - \sin^4 x\right)$$

$$h(x) = \tfrac{1}{2} \int x \sin 2x - \int \sin^2 x \, dx + \int \sin^4 x \, dx + \int \sin^4 x \, dx$$

✓ ✓

$$\sin^4 x = \sin^2 x \, \sin^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$\Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\Rightarrow \int \sin^4 x \, dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \int \frac{1 - 2\cos 2x + \cos^2 2x}{4} \, dx$$

$$= \frac{1}{4}\left(x - \boxed{\int \cos 2x \cdot 2 \, dx} + \frac{1}{2} \boxed{\int \cos^2 2x \cdot 2 dx} \right)$$

$\sin t \cos t = \frac{1}{2} \sin 2t$ $\qquad \sin 2x$ $\qquad \frac{1}{2} 2x + \frac{1}{4} \sin(2\cdot(2x))$

$$= \frac{1}{4}\left(x - \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x \right) + C$$

$\frac{1}{2} 2x + \frac{1}{2} \sin 2x \cos 2x$

$$= \frac{1}{4}\left(\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x \right) + C$$

$\frac{1}{4} \sin 2x \cos 2x$

$$\int \cos^2 x \cdot \cos x \cdot dx = \int (1 - \sin^2 x) \cos x \, dx$$

$$\int f + g = \int f + \int g$$

$$= (-\sin x) - \int (\sin x)^2 \cdot \cos x \, dx$$

$(\,)'$

$$(f \cdot g \cdot h)' = f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$$

$$(\cos x \cdot \cos x \cdot \sin x)' = (-\sin x) \cdot \cos x \cdot \sin x + \cos x\,(-\sin x)\,\sin x + \cos x \, \cos x \, \cos x$$

$$(\cos^2 x \sin x)' = -\sin^2 x \cos x - \sin^2 \cos x + \cos^3 x$$

$$\cos^2 x \sin x + C = -2 \int \sin^2 x \cos x \, dx + \int \cos^3 x \, dx$$

$$\cos^3 x = \cos^2 x \cdot \cos x$$

$$= (1 - \sin^2 x) \cdot \cos x \qquad (\sin x)^2 \cdot (\sin x)'$$

$$= \cos x - \sin^2 x \cos x$$

$$\int \cos^3 x \, dx = \sin x - \frac{1}{3} (\sin x)^3 + C$$

$$(f\cdot g)' = f'g + fg'$$

$$(\cos^2 x \sin x)' = 2\cos x\cdot(-\sin x)\cdot \sin x + \cos^2 x\cdot \cos x$$

$$\cos^2 x \sin x = -2\int \sin^2 x \cos x \, dx + \int \cos^3 x \, dx$$

$$= -2\int (1-\cos^2 x)\cos x \, dx + \int \cos^3 x \, dx$$

$$= -2\int \cos x \, dx + 2\int \cos^3 x \, dx + \int \cos^3 x \, dx$$

$$= -2\cdot \sin x + 3\int \cos^3 x \, dx$$

$$\Rightarrow \int \cos^3 x \, dx = \frac{2}{3}\sin x + \frac{1}{3}\sin x \cos^2 x + C$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$= 1 - \sin^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$\Rightarrow \boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$\int \sin^4 x \, dx = \int \sin^2 x \, \sin^2 x \, dx = \int \frac{(1-\cos 2x)(1-\cos 2x)}{4} dx$$

$$= \int \frac{1 - 2\cos(2x) + \cos^2(2x)}{4} dx$$