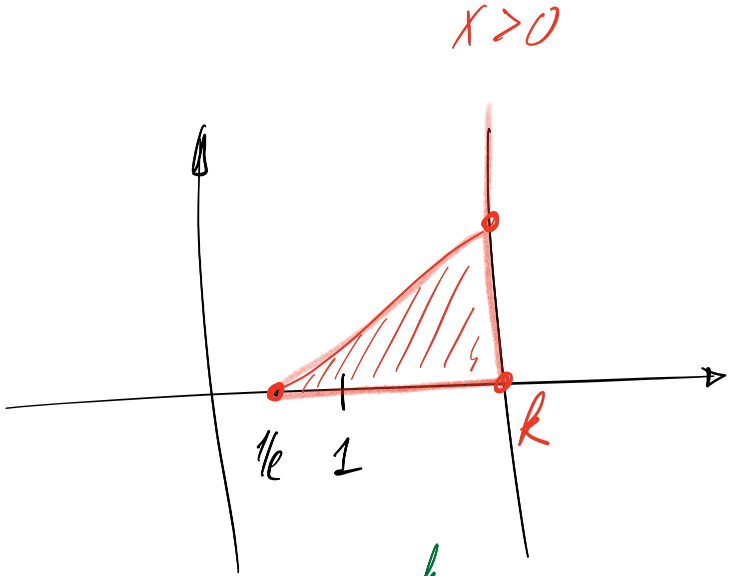
$$f(x) = \frac{(1+\ln(x))^2}{x}$$



$y = 0$

$x = k > 1$

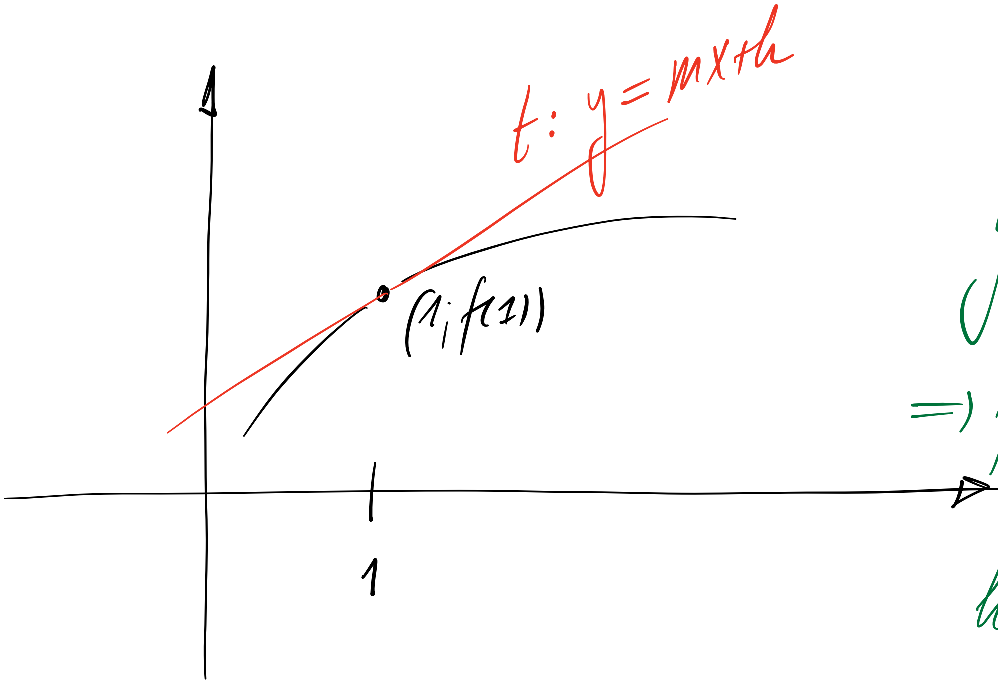
$f(x) = 0$ si $1 + \ln(x) = 0$

$\ln(x) = -1$

$x = e^{-1} = 1/e$

$$\int_{1/e}^{k} \frac{(1+\ln(x))^2}{x}\,dx$$

$$\int (\underbrace{1+\ln(x)}_{(\ )'})^2 \cdot \frac{1}{x}\,dx$$



$$(a; f(a)) = (1; f(1))$$

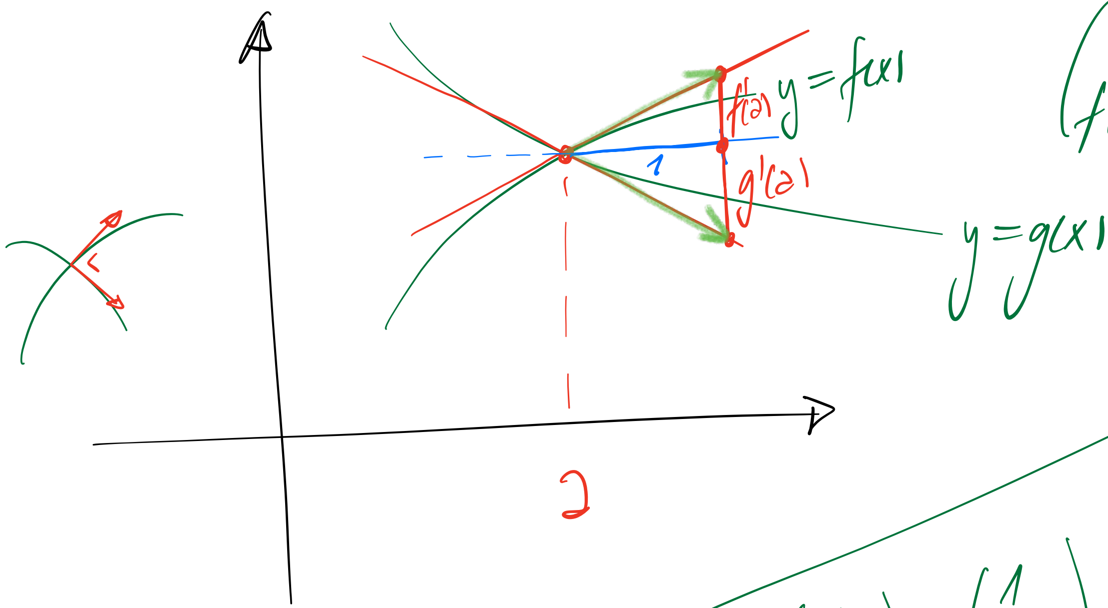
$$y = f'(a)\cdot x + h$$

$$\Rightarrow f(a) = f'(a)\cdot a + h$$

$$h = f(a) - f'(a)\cdot a$$

$$\Rightarrow t:\ y = f'(a)\cdot x + f(a) - f'(a)\cdot a$$

$$\boxed{y - f(a) = f'(a)\,(x-a)}$$



$$\begin{pmatrix}1\\f'(a)\end{pmatrix} \perp \begin{pmatrix}1\\g'(a)\end{pmatrix}$$

$$\begin{pmatrix}1\\f'(a)\end{pmatrix}\cdot\begin{pmatrix}1\\g'(a)\end{pmatrix}=0 \Leftrightarrow 1+f'(a)g'(a)=0$$

$$\Leftrightarrow f'(a) = -\frac{1}{g'(a)}$$

$$2^x = e^{\ln(2^x)} = \boxed{e^{x \cdot \ln(2)}}$$

définition de 2^x

$$e^{\ln(t)} = t$$

$$\ln(a^b) = b\ln(a)$$

$$\left(2^x\right)' = \left(e^{x\ln(2)}\right)'$$

$$= e^{x\ln(2)} \cdot \ln(2)$$

$$= \ln(2) \cdot 2^x$$

$$\int 2^{-2x} dx = \int e^{-2x \cdot \ln(2)} dx$$

$$= \frac{1}{-2\ln(2)} \int e^{(-2\ln(2)) \cdot x} \cdot (-2\ln(2)) \cdot dx$$

$$= \frac{1}{-2\ln(2)} \cdot e^{-2\ln(2) \cdot x} + C$$

$$= \frac{2^{-2x}}{-2\ln(2)} = -\frac{1}{\ln(2)} \cdot 2^{-2x-1}$$

$$f(x) = \boxed{\frac{\boxed{(1+\ln(x))^2}}{x}} = \frac{u(x)}{v(x)}$$

$$\frac{0}{A} = 0 \qquad \cancel{\frac{B}{0}}$$

$$f(x) = 0 \Leftrightarrow \frac{u(x)}{v(x)} = 0 \Leftrightarrow u(x) \text{ et } v(x) \neq 0$$

Ainsi, $(1+\ln(x))^2 = 0$

$$\boxed{T^2 = 0 \Leftrightarrow T = 0}$$

$$1 + \ln(x) = 0$$

$$\ln(x) = -1$$

$\ln(x) = u \Leftrightarrow x = e^u$

$$x = e^{-1} = 1/e \neq 0 \quad \text{car} \quad v(1/e) = 1/e \simeq 0{,}37$$

$$V = \{ a\cos x + b \mid a, b \in \mathbb{R},\ x \in [0; 2\pi] \}$$

Exemples de vecteurs :

$$u = \cos x - 2$$

$$u : [0; 2\pi] \to \mathbb{R}$$
$$x \mapsto \cos x - 2$$

$$v = 5\cos x + 4$$

$$z = \cos x$$

vecteur nul : $a = b = 0$

dans V — dans $\mathbb{R}$

$$(a\cos x + b) + (c\cos x + d) = \underbrace{(a+c)}_{a'}\cos x + \underbrace{(b+d)}_{b'}$$

$$= a'\cos x + b' \in V$$

$$u = a\cos x + b \qquad -u = (-a)\cos x - b$$

$$u + (-u) = (a + (-a))\cos x + (b - b) = 0 \cdot \cos x + 0$$
$$= 0$$

$a\cos x + b \in V \qquad k \in \mathbb{R}$

$$k \cdot (a\cos x + b) = \underbrace{(ak)}_{a'}\cos x + \underbrace{(bk)}_{b'} \in V \quad ✓$$

$$1\,(a\cos x + b) = a\cos x + b \quad ✓$$

$$(k+l)(a\cos x + b) \overset{?}{=} k(a\cos x + b) + l(a\cos x + b) \quad ✓$$

$$= (k+l)\cdot a \cdot \cos x + (k+l)\,b$$

$$= k\cdot a\cos x + l\cdot a\cos x + kb + lb$$

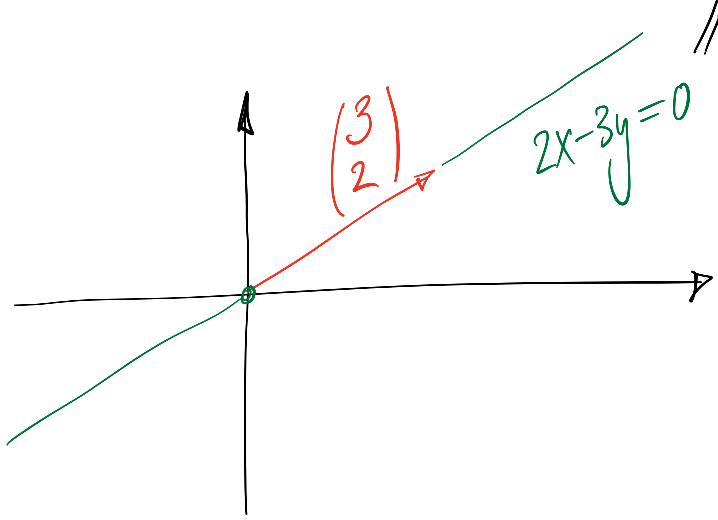
$$= ka\cos x + kb + la\cos x + lb$$

$$= k(a\cos x + b) + l(a\cos x + b)$$

$$k\big((a\cos x + b) + (c\cos x + d)\big) = k(a\cos x + b) + k(c\cos x + d)$$

Sous-espace vectoriel

$$\mathbb{R}^2 = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \,\middle|\, a,b \in \mathbb{R} \right\}$$



$$U = \left\{ k\begin{pmatrix} 3 \\ 2 \end{pmatrix} \,\middle|\, k \in \mathbb{R} \right\}$$

U est un sous-espace de $\mathbb{R}^2$

- $\boxed{0 \in U}$ $\left(\text{OK, car } \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0 \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{ ou } 2\cdot 0 - 3\cdot 0 = 0\right)$ ✓

  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

- $\boxed{u, v \in U \Rightarrow au + bv \in U}$

$$u = k\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$v = l\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$ak\begin{pmatrix} 3 \\ 2 \end{pmatrix} + bl\begin{pmatrix} 3 \\ 2 \end{pmatrix} =$$

$$\underbrace{(ak + bl)}_{\lambda \in \mathbb{R}}\begin{pmatrix} 3 \\ 2 \end{pmatrix} \in U$$