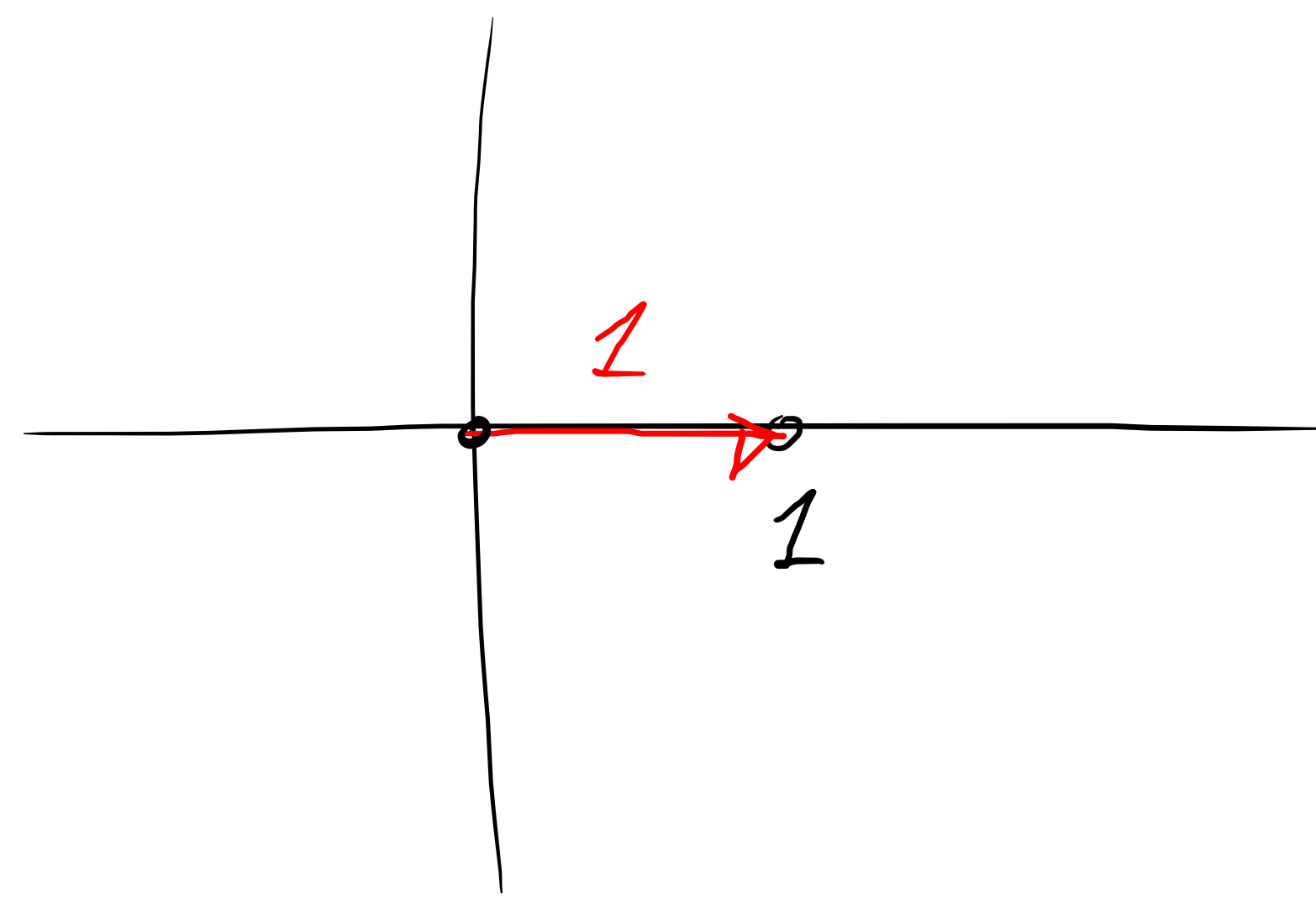


Racines n-èmes dans \mathbb{C}

Exemple: $z^3 = 1$

z_1, z_2, z_3

$z_1 = 1 + 0i$



$$\cos \alpha = \cos \beta$$

$$\alpha = \beta + k2\pi$$

$$\cos 3\theta = \cos 0$$

$$\sin 3\theta = \sin 0$$

$$[r; \theta]^3 = [1; 0]$$

$$[r^3; 3\theta] = [1; 0]$$

$$r^3(\cos 3\theta + i \sin 3\theta) = 1 \cdot (\cos 0 + i \sin 0)$$

$$r^3 = 1 \Leftrightarrow r = 1 = \sqrt[3]{1}$$

$$3\theta = 0 + k \cdot 2\pi$$

$$k \in \mathbb{Z}_3$$

$$\theta = \frac{0 + k \cdot 2\pi}{3}$$

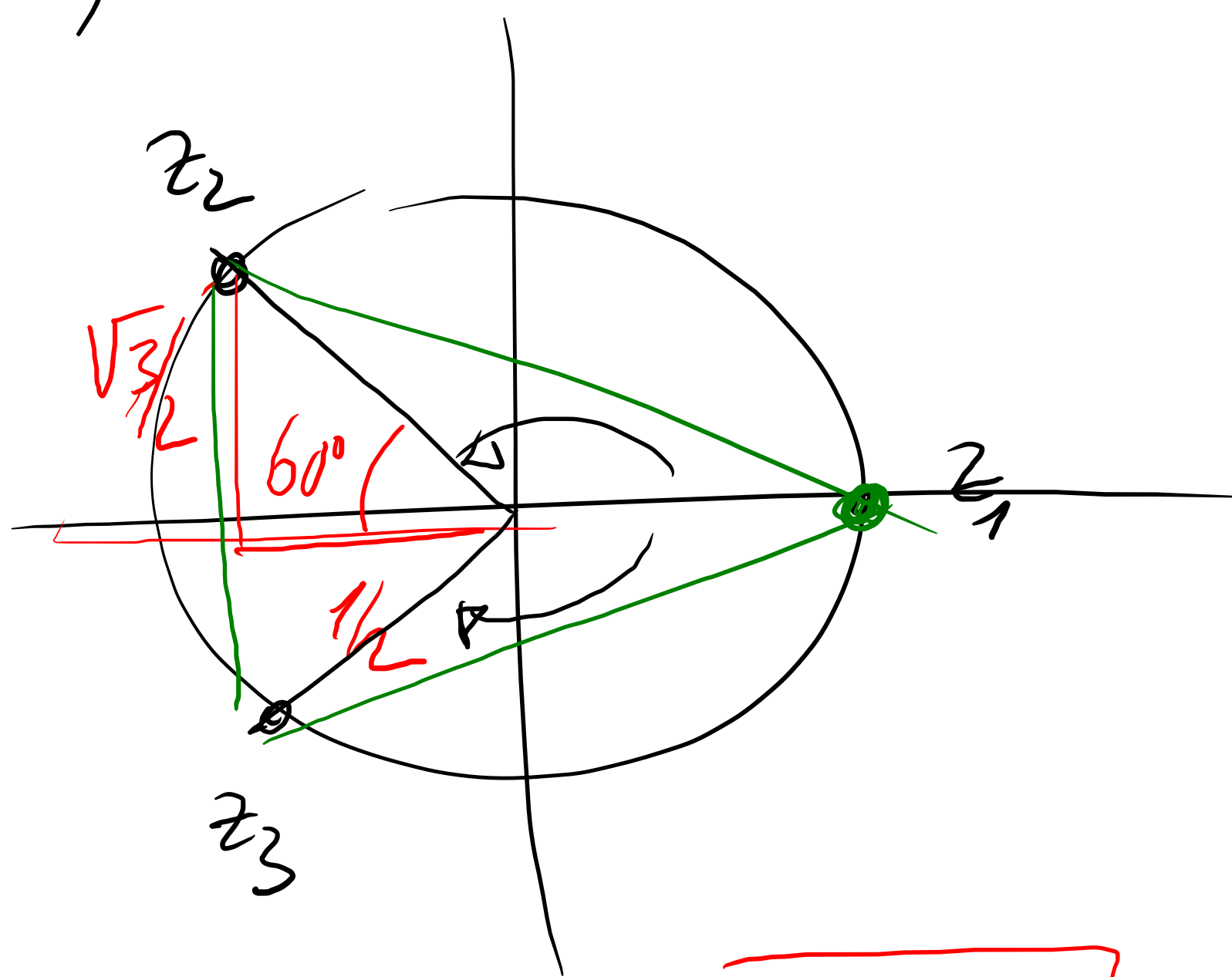
$$\frac{0 \cdot 2\pi}{3} = 0 \Rightarrow z_1 = 1 + 0i$$

$$\frac{1 \cdot 2\pi}{3} = \frac{2\pi}{3} \Rightarrow z_2 = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$$

$$= -\frac{1}{2} + i \frac{\sqrt{3}}{2}$$

$$\frac{2 \cdot 2\pi}{3} = \frac{4\pi}{3} \Rightarrow z_3 = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$$

$$= -\frac{1}{2} - i \frac{\sqrt{3}}{2}$$

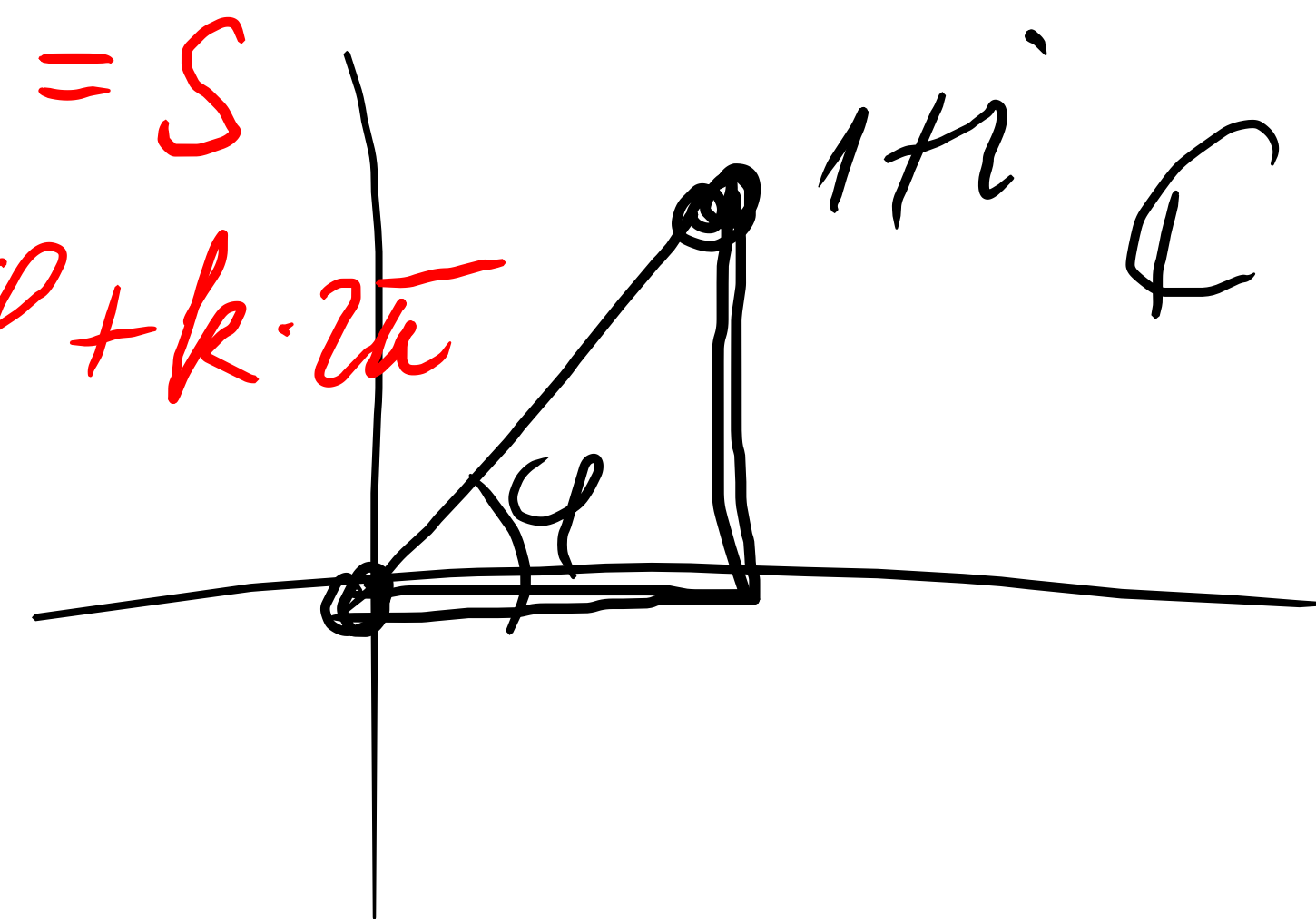


1.2.8

$$[r; \theta]^4 = [s; \varphi]$$

$$r^4 = s$$

$$4\theta = \varphi + k \cdot 2\pi$$



$$1+i = \left[\sqrt{2}, \frac{\pi}{4} \right]$$

$$= [\sqrt{2}; \varphi]$$

$$2) |1+i| = \sqrt{2} = s$$

$$\varphi = \frac{\pi}{4}$$

$$[r^4; 4\theta] = [r; \theta]^4 = 1+i = [s; \varphi] \Rightarrow$$

$$r = \sqrt[8]{2}$$

$$\Leftrightarrow r^4 = \sqrt{2}$$

$$4\theta = \varphi + k \cdot 2\pi$$

$$\theta = \frac{\varphi + k \cdot 2\pi}{4}$$

$$\theta = \frac{\frac{\pi}{4} + k \cdot 2\pi}{4}$$

$$r = \sqrt[4]{\sqrt{2}} = (2^{1/2})^{1/4} = 2^{1/8}$$

$$k = 0, 1, 2, 3$$

$$w^2 = z$$

$$|w|^2 = |z|$$

$$a^2 + b^2 = |z|$$

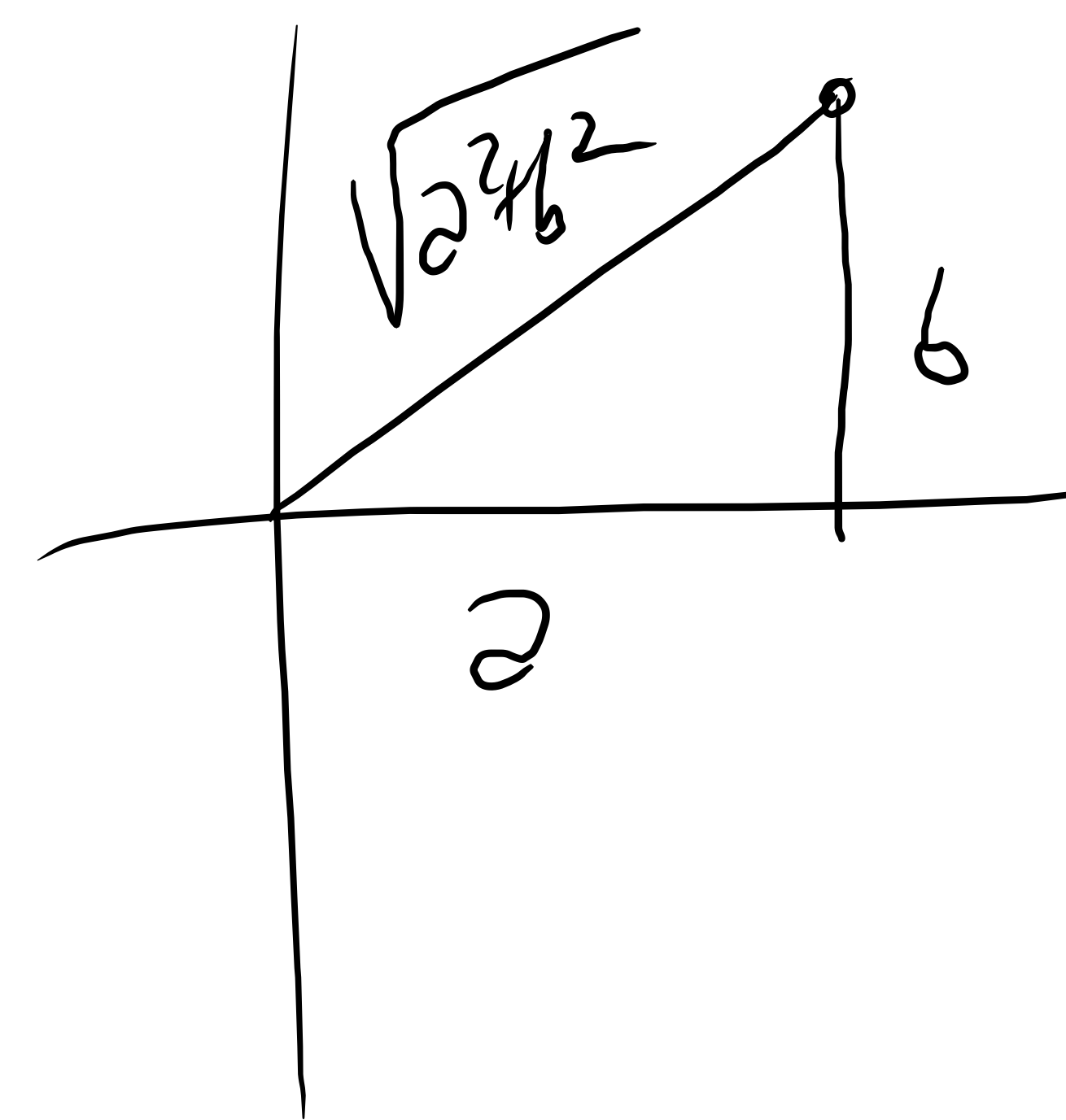
$$|-7 + 24i| = \sqrt{49 + 576} = \sqrt{625} = 25$$

$$w^2 = -7 + 24i$$

$$|w|^2 = a^2 + b^2$$

$$w = a + bi$$

$$|w| =$$



$$\begin{cases} a^2 + b^2 = 25 \\ a^2 - b^2 = -7 \\ 2ab = 24 \end{cases} \quad a, b \in \mathbb{R}$$

$$w^2 = a^2 + 2abi + (bi)^2$$

$$= \boxed{a^2 - b^2} + \boxed{2abi} = -7 + 24i = z$$

$$2a^2 = 18 \mid a^2 = 9 \mid a = \pm 3$$

$$\begin{cases} a = 3 & b = 4 \\ a = -3 & b = -4 \end{cases}$$

$$z = w^2$$

$$u^2 = w$$

$$3 + 4i$$

$$-3 - 4i$$

u