$$\boxed{\begin{aligned} x_0 &= 1 \\ x_{n+1} &= \frac{1}{1+x_n} \end{aligned}}$$

$$= \frac{1}{1+\frac{1}{1+x_{n-1}}}$$

$$= \frac{1}{1+\frac{1}{1+\frac{1}{1+x_{n-2}}}}$$

$$u = \frac{1}{1+u}$$

$$u^2+u-1=0$$

$$u = \frac{-1 \pm \sqrt{5}}{2}$$

$$\frac{1}{2} \qquad \frac{2}{3} \qquad \frac{3}{5} \qquad \frac{5}{8} \qquad \frac{8}{13}$$

$$\frac{1}{n^2+1} \xrightarrow{n\to\infty} 0$$

Prenves à partir de la déf.

$$\frac{2n^2}{n^2+1} \xrightarrow{n\to\infty} 2 \qquad \frac{n}{n+1} \xrightarrow{n\to\infty} 1$$

Déf: $\lim u_n = a \Leftrightarrow \forall \varepsilon>0 \ \exists N_\varepsilon$ tq. $n \geq N_\varepsilon \Rightarrow |u_n - a| < \varepsilon$

Soit $\varepsilon>0$

preuve: $\left|\frac{1}{n^2+1} - 0\right| < \varepsilon \qquad n\in\mathbb{N}$

suite

limite

très vrai si $\varepsilon > 1$

$\Leftrightarrow \left|\frac{1}{n^2+1}\right| < \varepsilon$

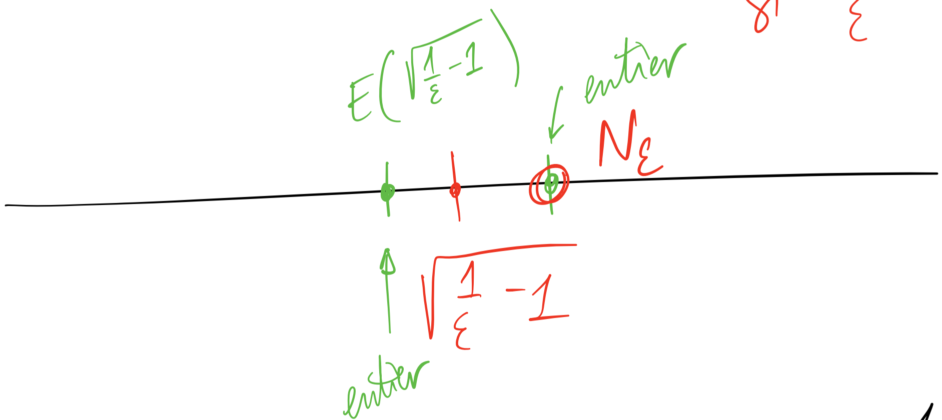
$n^2+1>0$ si $n\in\mathbb{N}$

$\Leftrightarrow \frac{1}{n^2+1} < \varepsilon \Leftrightarrow \frac{1}{\varepsilon} < n^2+1$

$\Leftrightarrow n^2 > \frac{1}{\varepsilon} - 1$

$$\Rightarrow N_\varepsilon = E\left(\sqrt{\frac{1}{\varepsilon}-1}\right)+1 \Leftrightarrow \boxed{n > \sqrt{\frac{1}{\varepsilon}-1}} \checkmark$$

si $\frac{1}{\varepsilon}-1 \geq 0 \Leftrightarrow \varepsilon \leq 1$




Donc si $n \geq N_\varepsilon \Rightarrow \left|\frac{1}{n^2+1} - 0\right| < \varepsilon$