Une famille de vecteurs

est libre / liée

linéairement indépendante / lin. dép.

(non) colinéaire (2 vecteurs)

(non) coplanaires (3 vecteurs)

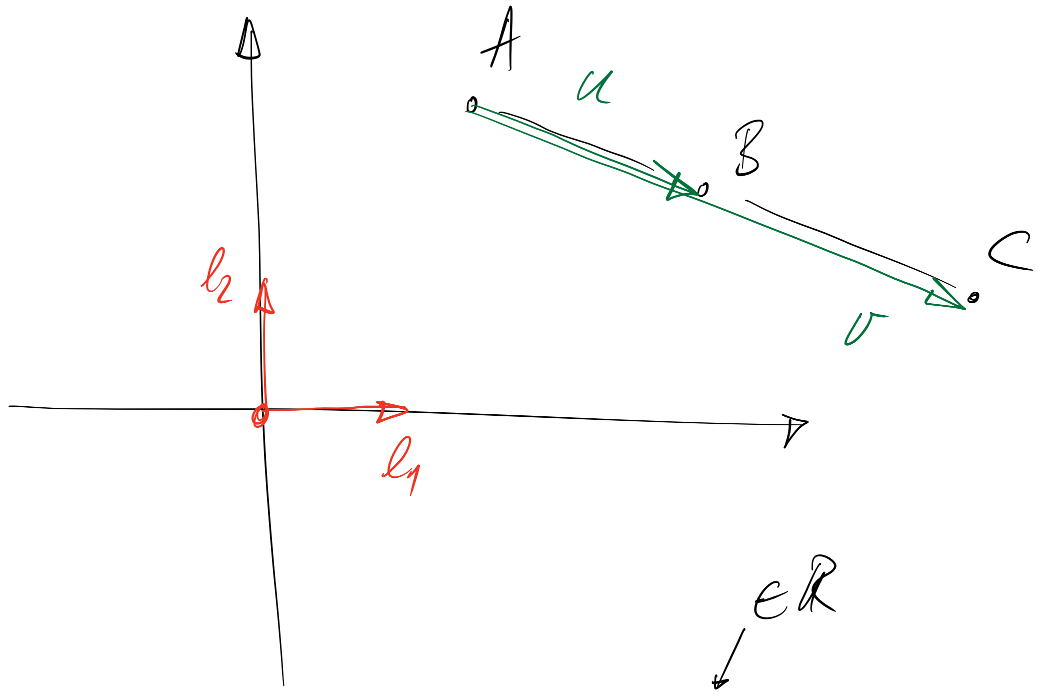
$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ et $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

sont $\boxed{\text{libres}}$ $\Leftrightarrow \det \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix} = \begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix}$

$= u_1 v_2 - u_2 v_1$

$\boxed{\neq 0}$

Alignement

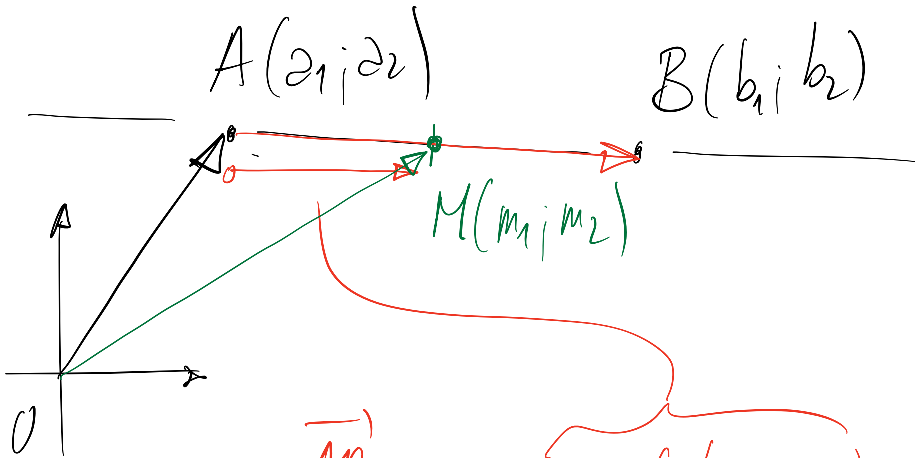


$\in \mathbb{R}$

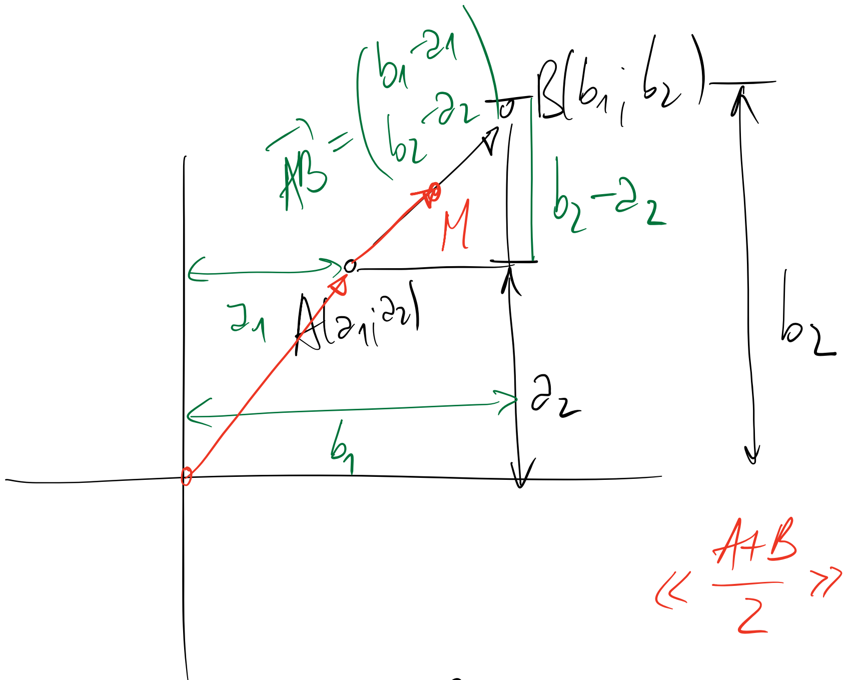
A, B, C sont alignés $\Leftrightarrow$ $u = k \cdot v$

$u = \overrightarrow{AB}$ $\quad v = \overrightarrow{AC}$

$\Leftrightarrow$ u et v liés



$$\vec{OA} + \frac{\vec{AB}}{} = \frac{1}{2}\begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix} + \begin{pmatrix} a_1 - 0 \\ a_2 - 0 \end{pmatrix}$$



$$\vec{OM} = \vec{OA} + \frac{1}{2}\vec{AB}$$

$$= \frac{1}{2}\begin{pmatrix} 2a_1 \\ 2a_2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix}$$

$$«\frac{A+B}{2}» = \frac{1}{2}\begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

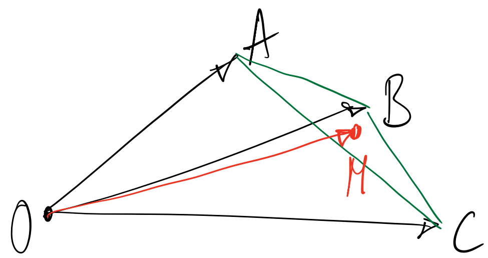
$$M = \frac{A+B}{2}$$

$\vec{AB} = \begin{pmatrix} -7 \\ -12 \end{pmatrix}$ $\quad \frac{1}{2}\vec{AB} = \begin{pmatrix} -3,5 \\ -6 \end{pmatrix}$ $\quad \vec{OA} + \frac{1}{2}\vec{AB}$ $\quad M = \begin{pmatrix} 1,5 \\ 2 \end{pmatrix}$ ✓

Example: $A(5;8)$ $\quad B(-2;-4)$ $\quad M = \left(\frac{5-2}{2}; \frac{8-4}{2}\right)$

$= (1,5; 2)$

De manière analogue :

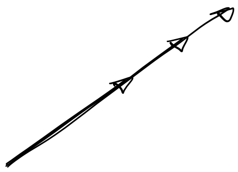


Soit A, B, C trois points

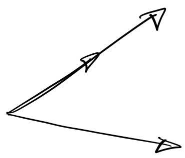
On pose $\overrightarrow{OM} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}}{3}$

« $M = \dfrac{A+B+C}{3}$ »

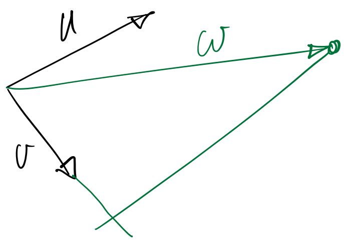
Coplanarité dans $\mathbb{R}^3$: u, v, w

① 

$u = k \cdot v = l \cdot w$ / colinéaires $\Rightarrow$ coplanaires

② 

$u = k \cdot v$ / w qcq / coplanaires

③ 

u, v libre

coplanaires $\Leftrightarrow$ $k \cdot u + l \cdot v = w$

Critères: u, v, w est libre

$$\Leftrightarrow \begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{vmatrix} = u_1 \cdot (v_2 w_3 - v_3 w_2) - u_2 \cdot (v_1 w_3 - v_3 w_1) + u_3 \cdot (v_1 w_2 - v_2 w_1) \neq 0$$

$\Leftrightarrow$ La seule solution de

$$x \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + y \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + z \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

et $x = y = z = 0$

$\boxed{1.3.6}$

$$\underbrace{k\vec{a}+l\vec{b}}_{\vec{u}} = \underbrace{x\vec{d}+y\vec{e}}_{\vec{u}}$$

$$y\begin{pmatrix} -35/15-2 \\ -14/15-1 \\ 10/15 \end{pmatrix} \quad \cdot(-15) \quad y\begin{pmatrix} 35+30 \\ 14+15 \\ -10 \end{pmatrix} \quad y\begin{pmatrix} 65 \\ 29 \\ -10 \end{pmatrix} \leftarrow \vec{u}'$$

$$-\frac{1}{15}y$$

$$\Leftrightarrow \quad k\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + l\begin{pmatrix} 0 \\ 8 \\ -5 \end{pmatrix} = x\begin{pmatrix} 35 \\ 14 \\ -10 \end{pmatrix} + y\begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \quad \begin{aligned} &\boxed{k = 35x - 2y} \\ -3k + 8l &= 14x - y \\ 2k - 5l &= -10x \end{aligned} \quad \Leftrightarrow \quad \left\{ \begin{aligned} k &= \left(-\frac{35}{15}-2\right)y \\ l &= \left(-\frac{16}{15}-\frac{4}{5}\right)y \\ x &= -\frac{1}{15}y \\ y &= y \end{aligned} \right.$$

$$\Leftrightarrow \quad \begin{aligned} -105x + 6y + 8l &= 14x - y \\ 70x - 4y - 5l &= -10x \end{aligned} \quad \Leftrightarrow \quad \left\{ \begin{aligned} k &= 35x - 2y \\ l &= 16x - \frac{4}{5}y \\ x &= -\frac{1}{15}y \\ y &= y \end{aligned} \right.$$

$$\Leftrightarrow \quad \begin{aligned} -119x + 7y &= -8l \\ 80x - 4y &= 5l \end{aligned}$$

$$\Leftrightarrow \quad (1)\ \frac{119}{8}x - \frac{7}{8}y = l$$

$$(2)\ 16x - \frac{4}{5}y = l \qquad \overset{(2)-(1)}{\Rightarrow} \quad \frac{9}{8}x + \left(\frac{7}{8} - \frac{4}{5}\right)y = 0 \qquad \frac{3}{40}$$

$$x = -\frac{8\cdot\cancel{3}}{\cancel{9}\cdot\cancel{40}_5}y = -\frac{1}{15}y \quad \Leftrightarrow \quad \frac{9}{8}x + \frac{3}{40}y = 0$$

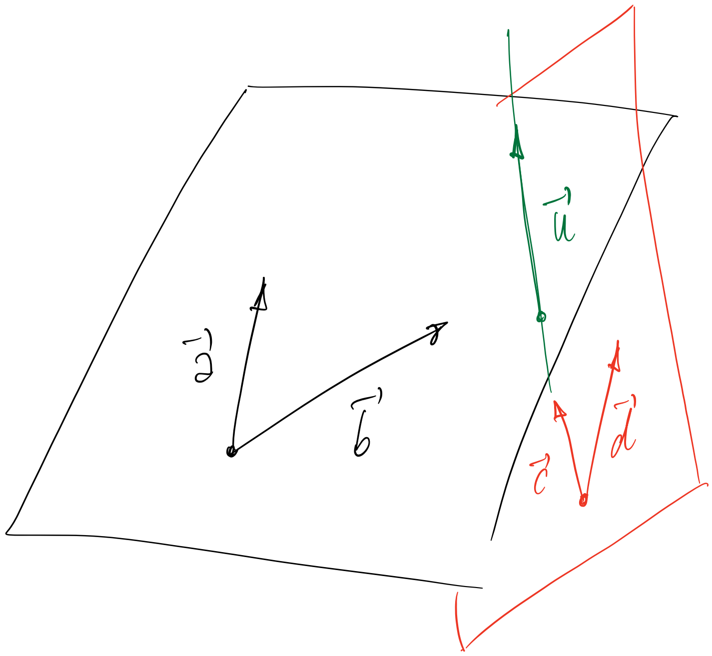
1.3.3

$$k\begin{pmatrix}7\\-2\end{pmatrix}+\lambda\begin{pmatrix}-3\\5\end{pmatrix}=\begin{pmatrix}0\\5\end{pmatrix}$$

$$\begin{cases}7k-3\lambda=0\\-2k+5\lambda=5\end{cases}$$

$$k=\frac{3}{7}\lambda$$

$$-\frac{6}{7}\lambda+5\lambda=5$$



$$\vec{u} = k \cdot \vec{a} + l\vec{b}$$

$$\vec{u} = x \cdot \vec{c} + y \cdot \vec{d}$$

$\boxed{2.3.14}$

$$x(x-2)(x+2)(x-3)(ax+b) = x(x-2)(x+2)(x-3)(2x-1)$$

$$\boxed{-3\cdot(-5)\cdot(-1)\cdot(-6)(-3a+b) = -630}$$

$$90(-3a+b) = -630$$

$$-3a+b = -7$$

$$\boxed{3a-b=7} \quad / \quad 3a-1+a=7$$

$$4a=8$$

$$\boxed{-1\cdot 3\cdot(-2)(a+b)=6}$$

$$a=2 \quad / \quad b=-1$$

$$\boxed{a+b=1} \quad / \quad b=1-a$$