$$N^1 = N = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$$

$$N^2 = N \cdot N = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$N^3 = N^2 \cdot N = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad N^n = 0 \quad \forall\, n \geq 2$$

$$U^1 = U = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$

$$U^2 = U \cdot U = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$$

$$U^3 = U^2 \cdot U = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad U^n = \begin{pmatrix} 1 & n \cdot a \\ 0 & 1 \end{pmatrix} \quad \forall\, n \geq 1$$

$$R^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$R^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R^3 = R^2 \cdot R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Longrightarrow \quad R^3 = R$$

$$R^4 = R^3 \cdot R = R \cdot R = R^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

En conclusion :

$$R^{2k+1} = R \qquad \text{pour } k = 0, 1, 2, 3, \ldots$$

$$R^{2k} = R^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{pour } k = 1, 2, 3, \ldots$$

$$T^1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$T^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$T^3 = T^2 \cdot T = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

$$T^4 = T^3 \cdot T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

On en déduit que

$$T^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$$