Burier 2023

Population	Les clients de Mr B.
Echantillon	La population en entier
Variable	L'heure à laquelle un client se présente.
Type	Quantitative / continue

x_i			f_i	F_i	$f_i \cdot x_i$
7,5	7h30	$n_1 = 1508$	$\frac{1508}{5200} = 0{,}29 = 29\%$	29% > 25% < 50% q_1	$0{,}29 \cdot 7{,}5$
8,5	8h30	$n_2 = 2184$	$\frac{2184}{5200} = 0{,}42 = 42\%$	71% > 50% q_2 < 75%	$0{,}42 \cdot 8{,}5$
9,5	9h30	$n_3 = 936$	$\frac{936}{5200} = 0{,}18 = 18\%$	89% > 75% q_3	$0{,}18 \cdot 9{,}5$
10,5	10h30	$n_4 = 572$	$\frac{572}{5200} = 0{,}11 = 11\%$	100%	$0{,}11 \cdot 10{,}5$
	Total →	5200 N	$1 = 100\%$		$\sum f_i x_i \simeq 8{,}61$

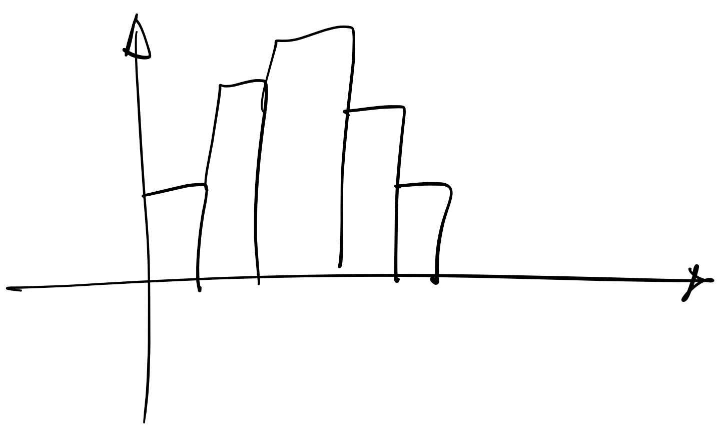
$$f_i = \frac{n_i}{N}$$

$$\text{moyenne} = \sum f_i \cdot x_i$$

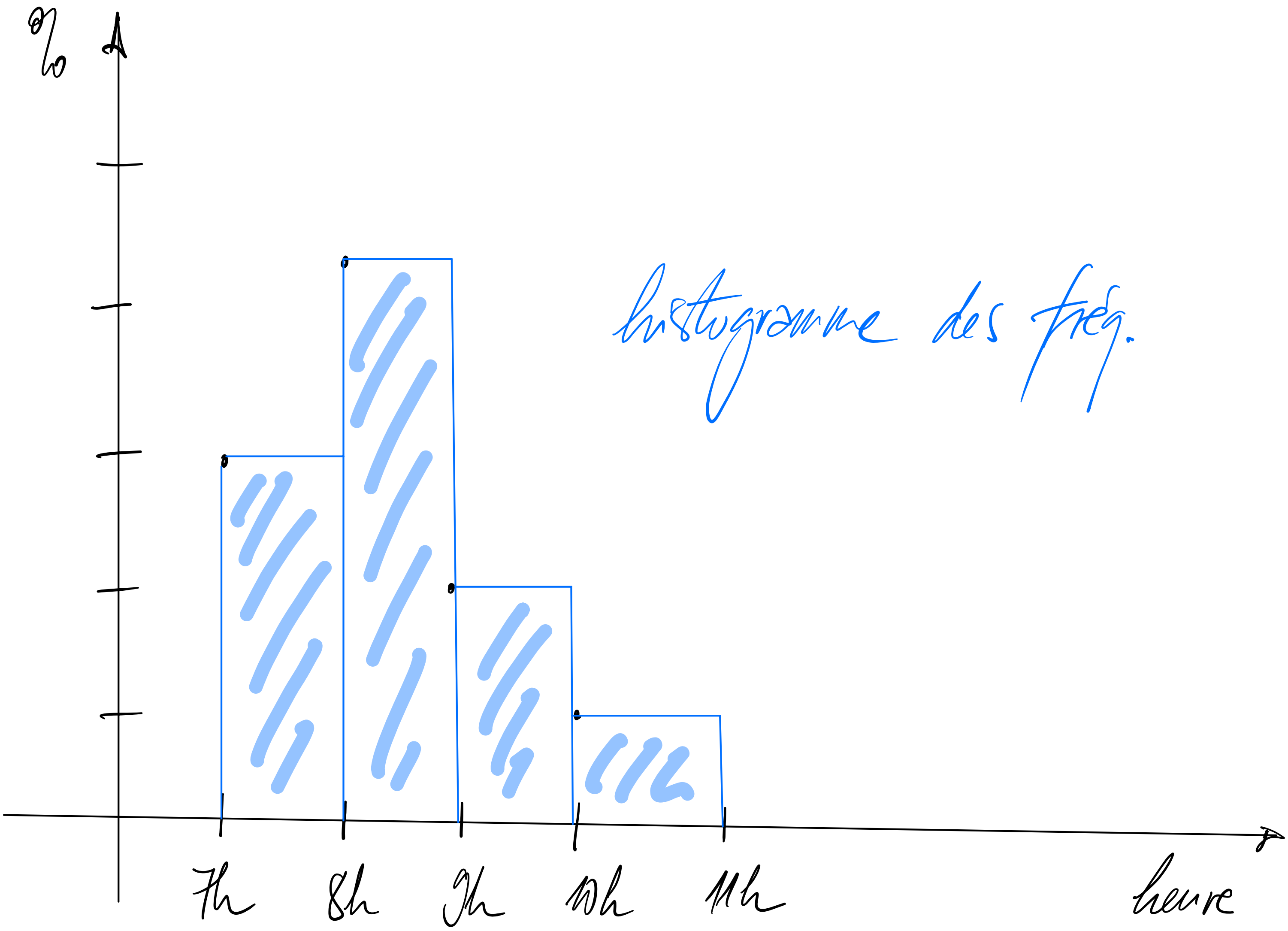
Question b)

8h36

Histogramme des fréq.

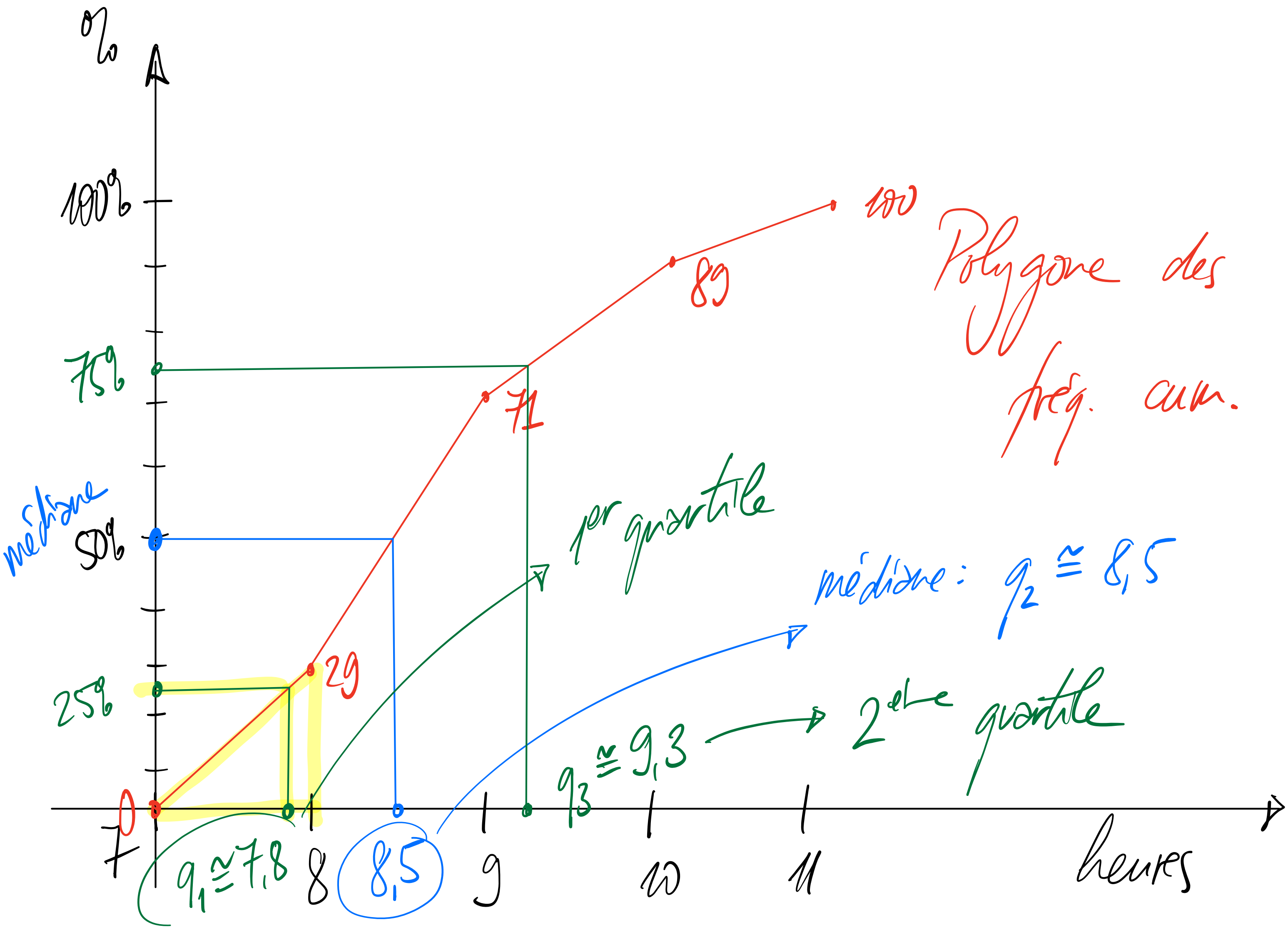


29/42/18/11 ← %



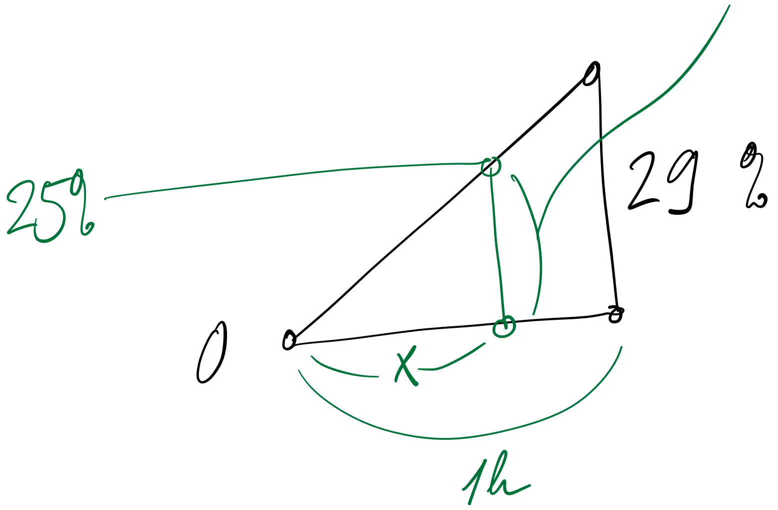
Polygone des fréq. cumulées

7h	0%
[7h ; 8h[	29%
[8h ; 9h[	71%
[9h ; 10h[	89%
[10h ; 11h[	100%



1er quartile

$25-0=25$



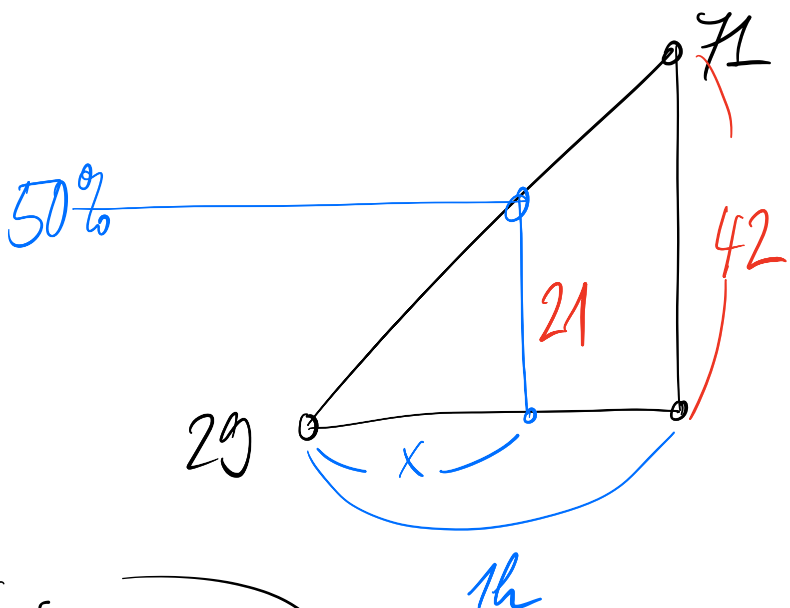
$$\frac{x}{1} = \frac{25}{29} \simeq 0,8621$$

$$\Rightarrow q_1 = 7+0,8621 \simeq 7,86$$

$$\simeq 7h52$$

2ème quartile / médiane

← Question c)

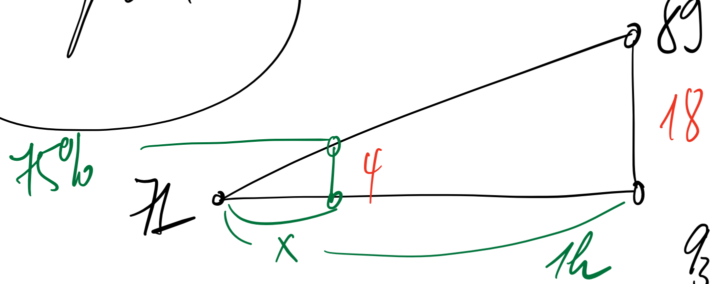


$$\frac{x}{1} = \frac{21}{42} = 0,5$$

$$q_2 = 8+0,5 = 8,5$$

$$= 8h30$$

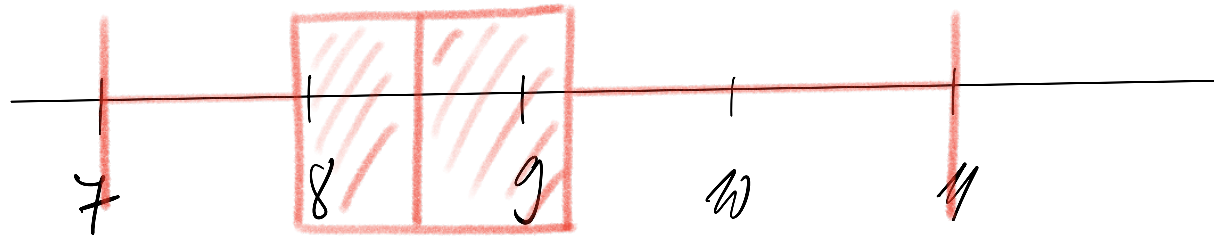
3ème quartile



$$\frac{x}{1} = \frac{4}{18} = \frac{2}{9} = 0,\overline{2}$$

$$q_3 = 9h13 \qquad q_3 = 9+0,\overline{2} \simeq 9,22$$

d)



boîte à moustaches ou box-plot

classes	F_i
[7h ; 8h[	29%
[8h ; 11h[	71%

e) cf. distribution

71% < 75%

L'affirmation est fausse

mode / moyenne / médiane / écart-type
quartiles

discret / continu (quantitatif)

	n_i	f_i	$f_i \cdot x_i$	$f_i \cdot (x_i - \bar{x})^2$
1	38	0,19	0,19	$0{,}19 \cdot (1 - 2{,}725)^2$
2	82	0,41	0,82	$0{,}41 \cdot (2 - 2{,}725)^2$
4	57	0,285	1,14	$0{,}285 \cdot (4 - 2{,}725)^2$
5	23	0,115	0,575	$0{,}115 \cdot (5 - 2{,}725)^2$
	200	1	2,725	1,84
	N			$\sqrt{1{,}84} \approx 1{,}36$

$\sum f_i x_i = \bar{x}$

$\sum f_i (x_i - \bar{x})^2 = V$ ← variance

$\sigma = \sqrt{V}$ ← écart-type

$f_i = \frac{n_i}{N}$

$$\bar{x} = 2{,}725$$

$$\bar{x} = \frac{1 \cdot 38 + 2 \cdot 82 + 4 \cdot 57 + 5 \cdot 23}{200} = 2{,}725$$

médiane et quartiles : cas discret

1	38
2	82
4	57
5	23
	200 = N

100

médiane

100

$x_{100} = 2$

$x_{101} = 2$

$q_2 \; \frac{x_{100} + x_{101}}{2} = 2$

50

50

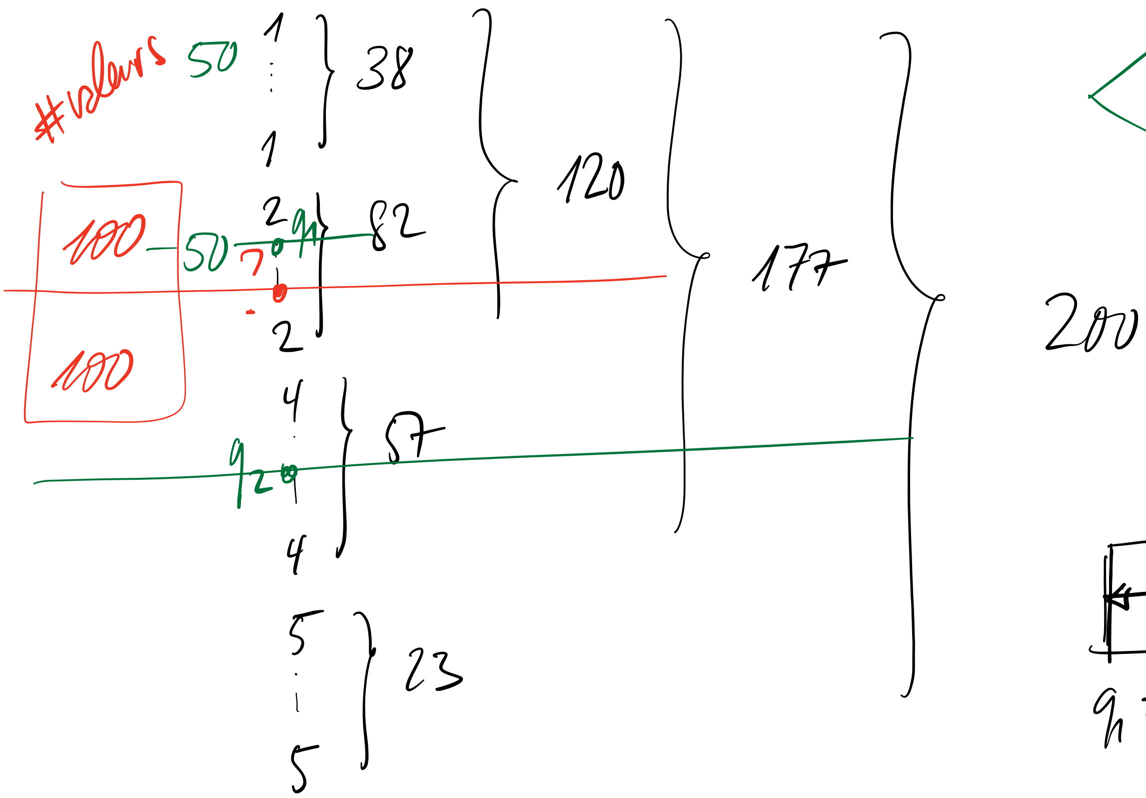
$q_1 = \frac{x_{50} + x_{51}}{2} = 2$

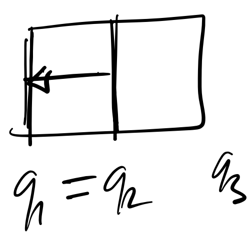
150

151

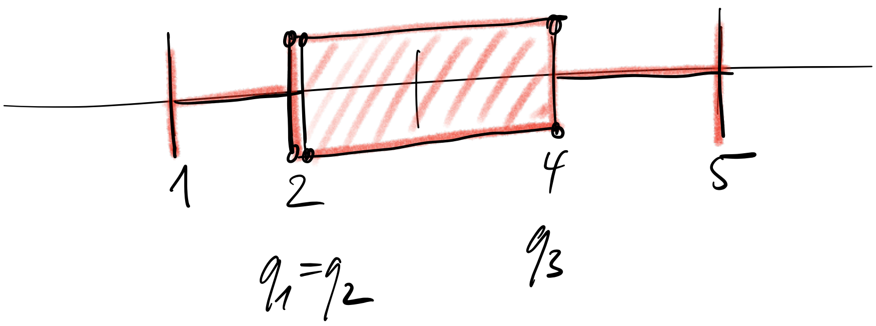
$q_3 = \frac{x_{150} + x_{151}}{2} = 4$

Visualisation des données :





Box plot :



$f_i \cdot x_i$

1	38		
2	82		
3			
4	57		
5	23		
12	1	0,005	$12 \cdot 0{,}005$

$N = 201$

important?

$\bar{x} = \sum f_i x_i$

Ecart-type : cas continu

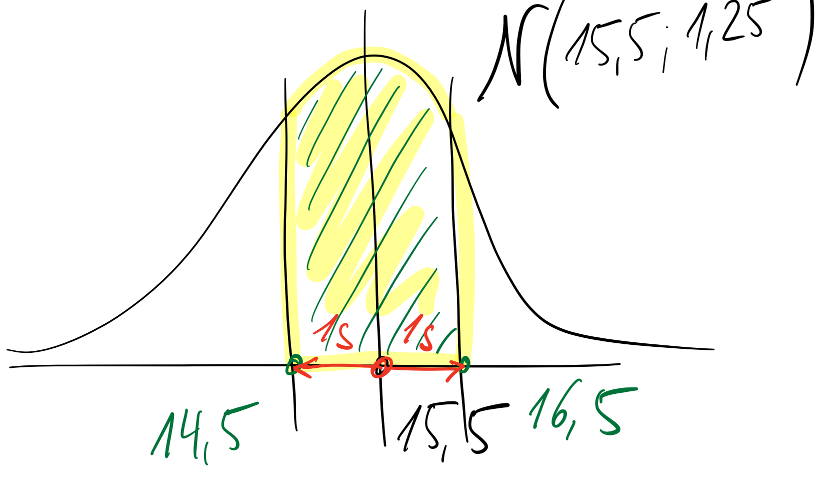
x_i (milieux)	classes	f_i	$f_i \cdot x_i$	$f_i(x_i - \overline{x})^2$
7,5	[7h00 ; 8h00[	0,29		$0{,}29 \cdot (7{,}5 - 8{,}61)^2 \simeq 0{,}3573$
8,5	[8h00 ; 9h00[	0,42		0,0051
9,5	[9h00 ; 10h00[	0,18		0,1426
10,5	[10h00 ; 11h00[	0,11		0,3929
			$\overline{x} = 8{,}61$	Variance : $\sum f_i(x_i - \overline{x})^2$ 0,8979

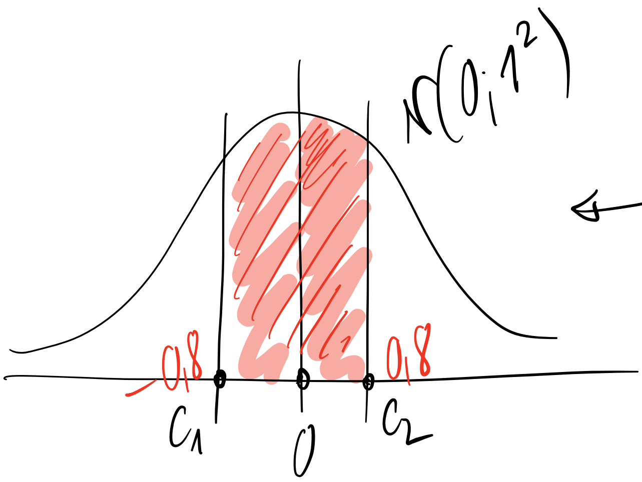
$\sqrt{0{,}8979} \simeq 0{,}9475$

$\sigma \simeq 0{,}95$

Loi normale

a) « A moins d'1s du temps moyen »

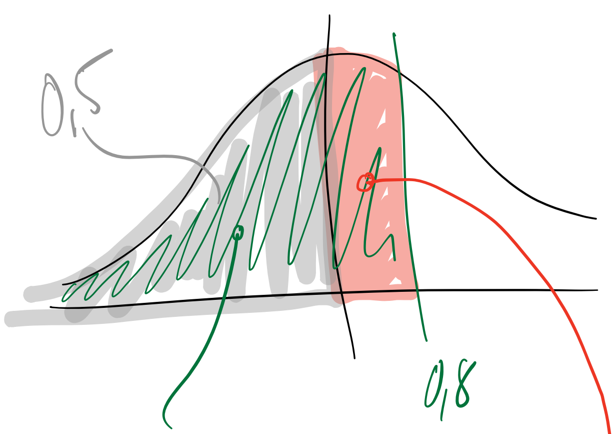




$$z = \frac{\text{Valeur} - \text{moyenne}}{\text{écart-type}}$$

$$c_1 = \frac{14{,}5 - 15{,}5}{1{,}25} = \frac{-1}{1{,}25} = -\frac{1}{\frac{5}{4}} = -\frac{4}{5} = -0{,}8$$

$$c_2 = \frac{16{,}5 - 15{,}5}{1{,}25} = 0{,}8$$

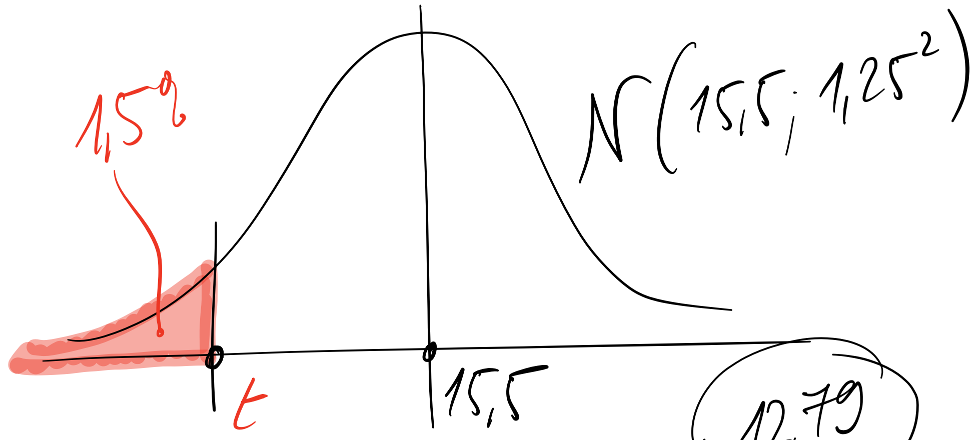


$p(Z < 0{,}8) = 0{,}7881$

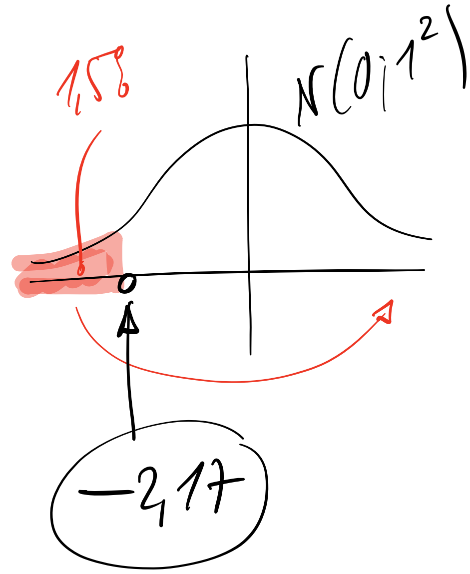
$p = 0{,}7881 - 0{,}5 = 0{,}2881$

$\Rightarrow p(14{,}5 \leq X \leq 16{,}5) = 2 \cdot 0{,}2881 = 0{,}5762$

b)



$t = 15{,}5 - 2{,}17 \cdot 1{,}25 = 12{,}7875 \approx 12{,}79$



$x = \text{moy} + z \cdot (\text{écart-type})$

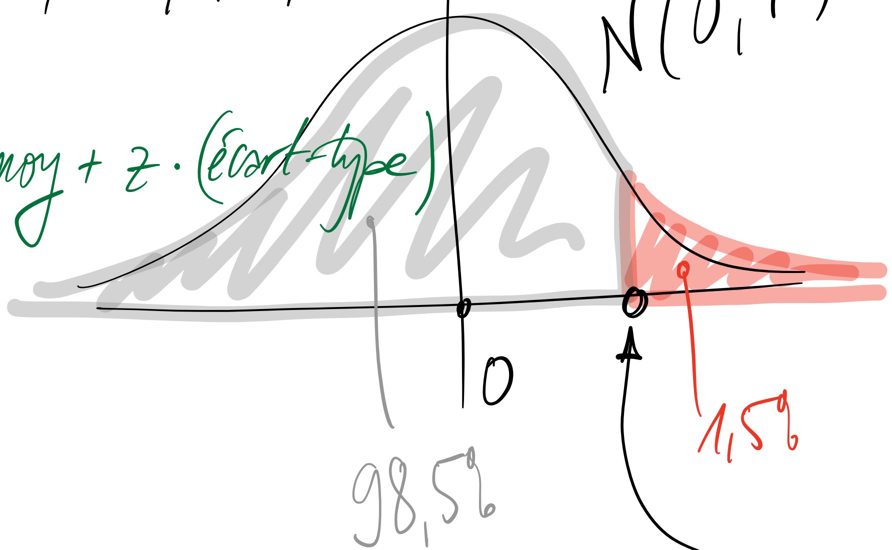


Table : $0{,}9850 \xrightarrow{?} 2{,}17$

c) Population : des recrues (modèle normal)

Echantillon : 50 recrues $\longrightarrow n = 50$

$n > 30$ } modèle normal pour la pop. $\Rightarrow$ loi des moyennes est normale

d) moyenne : 15,5 s (pop) TCL

écart-type : pop. grande

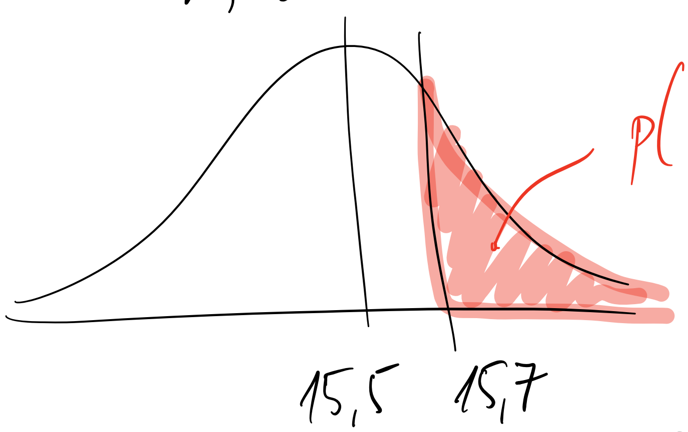
écart-type pop.

$$\frac{1{,}25}{\sqrt{50}} \simeq 0{,}17678$$

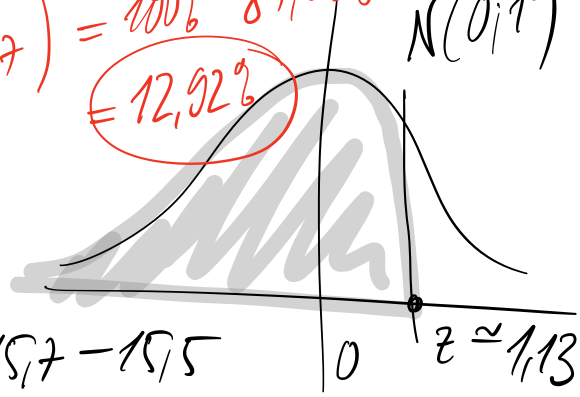
n, taille de l'échantillon

$N(15{,}5\,;\,0{,}17678^2)$

e)



$P(X > 15{,}7) = 100\% - 87{,}08\% = 12{,}92\%$

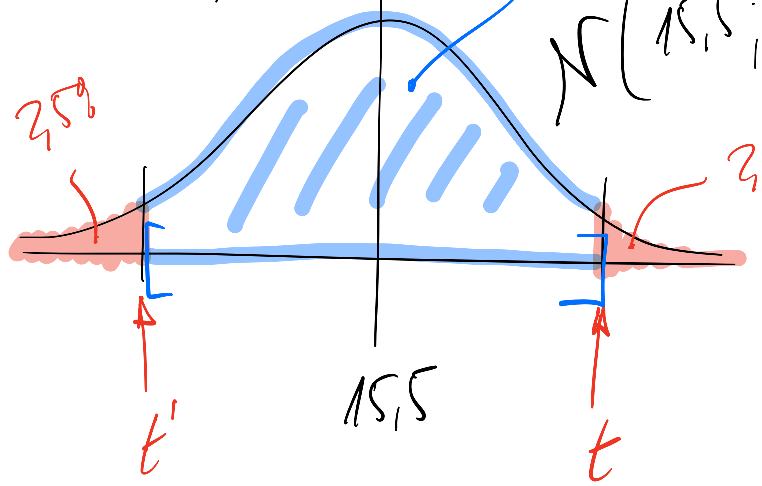


$$z = \frac{15{,}7 - 15{,}5}{0{,}17678}$$

$P(z < 1,13) \simeq 0,8708$

f)

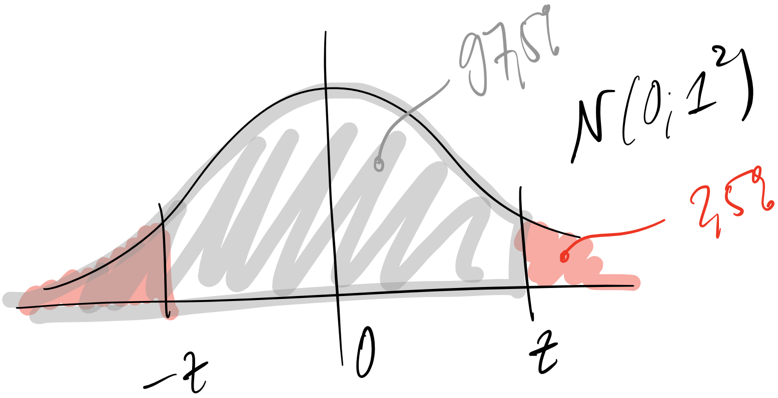
$N(15,5 ; 0,17678^2)$



$t = 15,5 + 1,96 \cdot 0,17678$

$t' = 15,5 - 1,96 \cdot 0,17678$

$N(0 ; 1^2)$



$t \simeq 15,85$

$t' \simeq 15,15$

Table : $0,9750 \longrightarrow 1,96$

$I = [15,15 ; 15,85]$

95% de la pop.

g) La taille de l'intervalle diminue.