$\boxed{4.2.5}$ Démontrer que $A \cdot B = B \cdot A$

$$A \cdot B = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix} \begin{pmatrix} \gamma' & \delta \\ -\delta & \gamma' \end{pmatrix} = \begin{pmatrix} \alpha \cdot \gamma' + \beta \cdot (-\delta) & \alpha\delta + \beta\gamma' \\ -\beta\gamma' - \alpha\delta & -\beta\delta + \alpha\gamma' \end{pmatrix}$$

$$= \begin{pmatrix} \alpha\gamma' - \beta\delta & \alpha\delta + \beta\gamma' \\ -\beta\gamma' - \alpha\delta & -\beta\delta + \alpha\gamma' \end{pmatrix}$$

$$\boxed{\alpha\delta + \beta\gamma' = \beta\gamma' + \alpha\delta}$$

$$B \cdot A = \begin{pmatrix} \gamma' & \delta \\ -\delta & \gamma' \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix} = \begin{pmatrix} \alpha\gamma' - \beta\delta & \beta\gamma' + \alpha\delta \\ -\alpha\delta - \beta\gamma' & -\beta\delta + \alpha\gamma' \end{pmatrix}$$

On voit que $AB = BA$.

$\boxed{4.2.6}$

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \quad \text{telle que} \quad \begin{matrix} A \cdot B \neq 0 \\ B \cdot A = 0 \end{matrix}$$

$$A \cdot B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ x & y \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} y & 0 \\ w & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{matrix} x \neq 0 \quad \text{ou} \quad \cancel{y \neq 0} \\ y = 0 \quad \text{et} \quad w = 0 \end{matrix}} \qquad \begin{matrix} x = 5 & w = 0 \\ z = -2 & y = 0 \end{matrix}$$

$$B = \begin{pmatrix} 5 & 0 \\ -2 & 0 \end{pmatrix}$$

$$A \cdot B \neq 0 \qquad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 5 & 0 \end{pmatrix} \leftarrow \text{non nulle}$$

$$B \cdot A = 0 \qquad \begin{pmatrix} 5 & 0 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\boxed{4.2.8}$

$$A = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \qquad {}^{t}A = \begin{pmatrix} x & z \\ y & w \end{pmatrix}$$

transpose

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad {}^{t}A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

✓

$$\begin{array}{cc} ① & ② \\ \end{array}$$
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{array}{c} ① \\ ② \end{array}\begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{pmatrix} = \begin{pmatrix} 21 & 18 & 15 \\ 51 & 44 & 37 \\ 81 & 70 & 59 \end{pmatrix}$$

$$\cancel{\begin{array}{cc} ① & ② \\ 1 & 2 \\ 3 & 4 \end{array} \quad \begin{array}{c} ① \\ ② \\ ③ \end{array}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}$$

$\boxed{1.1.1}$

a) $$A \cdot B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 3\cdot 0+2\cdot 1 & 14 & 12 \\ 4 & 8 & 14 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 14 & 12 \\ 4 & 8 & 14 \end{pmatrix}$$

$$\cancel{B \cdot A} = \begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{pmatrix} 0 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix} \end{array} \begin{array}{c} ① \\ ② \end{array}\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

Calcul impossible

$\boxed{\text{TE du } 02/04/2026}$

4.2.1 à 4.2.8 (Sauf 4.2.7)

1.1.1

1.1.2

1.1.10