

$$\int \frac{1}{2x-5} dx = \int \frac{1}{2x-5} \cdot 2 \cdot \frac{1}{2} dx$$

$$= \frac{1}{2} \int \underbrace{\frac{1}{2x-5}}_{()'} \cdot 2 dx = \frac{1}{2} \int \frac{1}{2x-5} \underbrace{(2x-5)'}_{()'} dx$$

$$= \frac{1}{2} \cdot \ln |2x-5| + C$$

$$\int \frac{5}{3x+2} dx = 5 \int \frac{1}{3x+2} \cdot 3 \cdot \frac{1}{3} dx$$

$$= 5 \cdot \frac{1}{3} \int \frac{1}{3x+2} \cdot \underbrace{(3x+2)'}_{()'} dx$$

$$= \frac{5}{3} \cdot \ln |3x+2| + C$$

$$\int \frac{x+2}{x-3} dx = \int \frac{\overbrace{x-3+5}^{+2}}{x-3} dx$$

$$= \int \frac{x-3}{x-3} dx + \int \frac{5}{x-3} dx$$

$$= \int 1 dx + 5 \int \frac{1}{x-3} dx$$

$(x-3)' = 1$

$$= x + 5 \ln |x-3| + C$$

$$1-x = -x+1$$

$$\begin{array}{r|l} 2x+1 & -x+1 \\ \hline 2x-2 & -2 \\ \hline 3 & \end{array}$$

$$2x+1 = (-2)(-x+1) + 3$$

$$\Rightarrow \int \frac{2x+1}{1-x} dx = \int \frac{(-2)(-x+1) + 3}{-x+1} dx$$

$$= \int \frac{(-2)(-x+1)}{-x+1} dx + \int \frac{3}{-x+1} dx$$

$$= -2 \int 1 dx + 3 \int \frac{1}{-x+1} dx$$

$$= -2x + 3 \int \underbrace{\frac{1}{-x+1}}_{()'} (-1) \cdot (-1) dx$$

$$= -2x - 3 \int \underbrace{\frac{1}{-x+1}}_A \cdot (-1) dx = -2x - 3 \ln|-x+1| + C$$

$$\begin{array}{r|l}
 x^2+2x+3 & x-1 \\
 \hline
 x^2-x & x+3 \\
 \hline
 3x+3 & \\
 3x-3 & \\
 \hline
 6 &
 \end{array}$$

$$\int \frac{x^2+2x+3}{x-1} dx = \int \frac{(x-1)(x+3) + 6}{x-1} dx$$

$$= \int \frac{(x-1)(x+3)}{x-1} dx + \int \frac{6}{x-1} dx$$

$$= \int (x+3) dx + 6 \int \frac{1}{x-1} dx \quad (x-1)' = 1$$

$$= \frac{1}{2}x^2 + 3x + 6 \ln|x-1| + C$$

$$\begin{array}{r|l}
 x^3 - x + 1 & x+1 \\
 \hline
 x^3 + x^2 & x^2 - x \\
 \hline
 -x^2 - x + 1 & \\
 -x^2 - x & \\
 \hline
 1 &
 \end{array}$$

$$\int \frac{x^3 - x + 1}{x+1} dx = \int \frac{(x+1)(x^2 - x) + 1}{x+1} dx$$

$$= \int (x^2 - x) dx + \int \frac{1}{x+1} dx$$

$$= \frac{1}{3} x^3 - \frac{1}{2} x^2 + \ln|x+1| + C$$

$$\begin{array}{r|l}
 2x^2+2x-3 & 2x+1 \\
 \hline
 2x^2+x & x+\frac{1}{2} \\
 \hline
 x-3 & \\
 x+\frac{1}{2} & \\
 \hline
 -\frac{7}{2} &
 \end{array}$$

$$\int \frac{2x^2+2x-3}{2x^2+x} dx = \int x + \frac{1}{2} dx - \frac{7}{2} \int \frac{1}{2x+1} dx$$

$$= \frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{2} \cdot \frac{1}{2} \cdot \int \frac{1}{2x+1} \cdot 2 dx$$

$$= \frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} \ln|2x+1| + C$$