$\boxed{1.3.15}$

3MS

a) $$\int_1^2 \frac{x}{x+6}\,dx = \int_1^2 \frac{x+6-6}{x+6}\,dx$$

$$= \int_1^2 \frac{x+6}{x+6} - \frac{6}{x+6}\,dx = \int_1^2 1\,dx - 6\int \frac{1}{x+6}\,dx$$

$$= x\Big|_1^2 - 6\ln|x+6|\Big|_1^2$$

$$= 2-1-6(\ln 8 - \ln 7)$$

$$= 1 - 6\ln(8/7) = 1+6\ln(7/8)$$

b) $$\int_0^4 (\sqrt{x}\cdot x + 2\sqrt{x})\,dx = \int_0^4 x^{3/2}\,dx + 2\int_0^4 \sqrt{x}\,dx$$

$$= \frac{1}{3/2+1}x^{3/2+1}\Big|_0^4 + 2\frac{1}{1/2+1}x^{1/2+1}\Big|_0^4$$

$$= \frac{2}{5}x^{5/2}\Big|_0^4 + \frac{4}{3}x^{3/2}\Big|_0^4$$

$\boxed{1.3.15}$ 3MS

$$= \frac{2}{5} 4^{5/2} + \frac{4}{3} 4^{3/2}$$

$$= \frac{2}{5}\left(\sqrt{4}\right)^5 + \frac{4}{3}\left(\sqrt{4}\right)^3$$

$$= \frac{2}{5} \cdot 2^5 + \frac{4}{3} \cdot 2^3 = \frac{64}{5} + \frac{32}{3} = \frac{192+160}{15}$$

$$= \frac{352}{15}$$

c) $$\frac{1}{6}(\sin x)^6 \Big|_0^{\pi/2} = \frac{1}{6}$$

d) $$\int_2^{\sqrt{20}} 3 \cdot \sqrt{\underbrace{x^2+5}} \cdot 2x \cdot \frac{1}{2}\, dx$$

$( )'$

$$= \frac{3}{2} \int_2^{\sqrt{20}} \sqrt{x^2+5} \cdot (x^2+5)'\, dx$$

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$$= \frac{3}{2}\int_2^{\sqrt{20}} (x^2+5)^{1/2} \cdot (x^2+5)' \, dx$$

$$= \underbrace{\frac{3}{2}\,\frac{1}{1/2+1}}^{1} (x^2+5)^{1/2+1} \Big|_2^{\sqrt{20}}$$

$$= (x^2+5)^{3/2} \Big|_2^{\sqrt{20}} = (20+5)^{3/2} - (4+5)^{3/2}$$

$$= (\sqrt{25})^3 - (\sqrt{9})^3$$

$$= 125 - 27 = 98$$

e) $$\int_0^3 \sqrt{9-x^2} \cdot x \cdot dx = -\frac{1}{2}\int_0^3 \sqrt{9-x^2} \cdot (-2x) \cdot dx$$

$$(9-x^2)' = 0 - 2x = -2x$$

$$= -\frac{1}{2}\int_0^3 \sqrt{9-x^2} \cdot (9-x^2)' \, dx$$

$\boxed{1.3.15}$

3MS

$$= -\frac{1}{2}\int_0^3 \left(9-x^2\right)^{1/2} \cdot \left(9-x^2\right)' dx$$

$$= -\frac{1}{2} \cdot \frac{1}{1/2+1}\left(9-x^2\right)^{1/2+1}\Bigg|_0^3$$

$$= -\frac{1}{2} \cdot \frac{2}{3}\left(9-x^2\right)^{3/2}\Bigg|_0^3$$

$$= -\frac{1}{3} \cdot \left(\sqrt{9-x^2}\right)^3\Bigg|_0^3$$

$$= -\frac{1}{3}\left(\left(\sqrt{9-9}\right)^3 - \left(\sqrt{9-0^2}\right)^3\right)$$

$$= -\frac{1}{3}\left(0 - 27\right) = \frac{27}{3} = 9$$

$\boxed{1.3.15}$

3MS

f) $\displaystyle\int_2^3 \frac{5x-2}{x^2-x}\,dx = \int_2^3 \left(\frac{2}{x}+\frac{3}{x-1}\right)dx$

En effet, $\displaystyle\frac{2}{x}+\frac{3}{x-1} = \frac{2\cdot(x-1)}{x\cdot(x-1)}+\frac{3\cdot x}{x(x-1)}$

$$= \frac{2x-2+3x}{x(x-1)} = \frac{5x-2}{x^2-x}$$

$$\Rightarrow \int_2^3 \frac{5x-2}{x^2-x}\,dx = 2\int_2^3 \frac{1}{x}\,dx + 3\int_2^3 \frac{1}{x-1}\,dx$$

$$= 2\ln|x|\Big|_2^3 + 3\ln|x-1|\Big|_2^3$$

$$= 2\ln\frac{3}{2}+3\ln 2 = \ln\frac{9}{4}+\ln 8$$

$$= \ln\left(\frac{9}{4}\cdot 8\right) = \ln 18$$