Exercice 1.2.11 c)

c) $\displaystyle\prod_{k=0}^{n} \cos(2^k\alpha) = \frac{\sin(2^{n+1}\alpha)}{2^{n+1}\sin(\alpha)}$

① vraie pour $n=0$: $\cos(\alpha)$ et $\displaystyle\frac{\sin(2\alpha)}{2\sin(\alpha)} = \frac{\cancel{2\sin(\alpha)}\cos(\alpha)}{\cancel{2\sin(\alpha)}}$

$= \cos(\alpha)$ ✓

② Supposons la relation vraie pour n et démontrons-la pour $n+1$.

$\underbrace{\cos(\alpha)\cdot\cos(2\alpha)\cdots\cos(2^n\alpha)}\cdot\cos(2^{n+1}\alpha) =$

$$\frac{\sin(2^{n+1}\alpha)}{2^{n+1}\sin(\alpha)}\cdot\cos(2^{n+1}\alpha) = \frac{\frac{1}{2}\sin(2\cdot 2^{n+1}\alpha)}{2^{n+1}\sin(\alpha)}$$

$\boxed{\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)} \Rightarrow \sin(\alpha)\cos(\alpha) = \frac{1}{2}\sin(2\alpha)$

$$= \frac{\sin(2^{n+2}\alpha)}{2^{n+2}\sin(\alpha)}$$