

$$\sum_{i=1}^n (2i-1)^2 = \sum_{i=1}^n (4i^2 - 4i + 1)$$

$$= \sum_{i=1}^n (4i^2) - \sum_{i=1}^n 4i + \sum_{i=1}^n 1$$

$$= 4 \sum_{i=1}^n i^2 - 4 \sum_{i=1}^n i + n$$

$$= 4 \cdot \frac{n(n+1)(2n+1)}{6} - 4 \cdot \frac{n(n+1)}{2} + n$$

$$= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n$$

$$= n \cdot \left(\frac{2(n+1)(2n+1) - 6(n+1) + 3}{3} \right)$$

$$= n \cdot \left(\frac{4n^2 + 2n + 4n + 2 - 6n - 6 + 3}{3} \right)$$

$$= n \cdot \frac{4n^2 - 1}{3} = \frac{n \cdot (2n - 1)(2n + 1)}{3}$$