d) $\boxed{n=1}$

$$\sum_{k=1}^{1} (-1)^{k+1} k^2 = (-1)^{1+1} \cdot 1^2 = 1$$

$$= (-1)^{1+1} \cdot \frac{1(1+1)}{2} = \frac{2}{2}$$

$\boxed{n ✓ \Rightarrow n+1 ✓}$

$$\sum_{k=1}^{n+1} (-1)^{k+1} \cdot k^2 = \sum_{k=1}^{n} (-1)^{k+1} \cdot k^2 + (-1)^{(n+1+1)} \cdot (n+1)^2$$

hyp. de réc.

$$= (-1)^{n+1} \cdot \frac{n(n+1)}{2} + (-1) \cdot (-1)^{n+1} \cdot (n+1)^2$$

$$= (-1)^{n+1} \cdot \left( \frac{n(n+1)}{2} - (n+1)^2 \right)$$

$$= (-1)^{n+1}\left(\frac{n^2+n-2(n^2+2n+1)}{2}\right)$$

$$= (-1)^{n+1}\left(\frac{n^2+n-2n^2-4n-2}{2}\right)$$

$$= (-1)^{n+1}\cdot(-1)\cdot\frac{n^2+3n+2}{2}$$

$$= (-1)^{n+2}\cdot\frac{(n+1)(n+2)}{2}$$

$$= (-1)^{(n+1+1)}\cdot\frac{(n+1)\cdot(n+1+1)}{2}$$

CQFD