

$$\begin{aligned} f'(x) &= x' \cdot e^x + x \cdot (e^x)' = e^x + x \cdot e^x \\ &= (1+x) e^x \end{aligned}$$

$$\begin{aligned} f''(x) &= (1+x)' e^x + (1+x)(e^x)' = e^x + (1+x) e^x \\ &= (2+x) e^x \end{aligned}$$

$$\begin{aligned} f^{(3)}(x) &= (2+x)' e^x + (2+x)(e^x)' = e^x + (2+x) e^x \\ &= (3+x) e^x \end{aligned}$$

On imagine assez facilement la suite :

$$f^{(n)}(x) = (n+x) e^x$$

pour n entier, $n \geq 1$.