$$
\begin{array}{r|l}
x^5 & x^3-1 \\ \cline{2-2}
x^5 - x^2 & x^2 \\ \cline{1-1}
x^2 &
\end{array}
\qquad \Rightarrow \frac{x^5}{x^3-1} = x^2 + \frac{x^2}{x^3-1}
$$

$$\Rightarrow \int \frac{x^5}{x^3-1}\,dx = \frac{1}{3}x^3 + \int \frac{x^2}{x^3-1}\,dx$$

$$\frac{x^2}{x^3-1} = \frac{x^2}{(x-1)(x^2+x+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+x+1}$$

(1) Multiplier par $x-1$; poser $x=1$:

$$\frac{x^2}{x^2+x+1} = a + \frac{(bx+c)(x-1)}{x^2+x+1} \Rightarrow \frac{1}{3} = a$$

(2) Poser $x=0$: $\quad 0 = -\frac{1}{3} + c \Rightarrow \frac{1}{3} = c$

(3) Multiplier par x ; passer à la limite $(x \to +\infty)$:

$$\frac{x^3}{x^3-1} = \frac{ax}{x-1} + \frac{bx^2+cx}{x^2+x+1} \xrightarrow{x\to+\infty} 1 = 1/3 + b \,/\, b = 2/3$$

$$\Rightarrow \frac{x^2}{x^3-1} = \frac{1}{3}\cdot\frac{1}{x-1} + \frac{2x+1}{3(x^2+x+1)}$$

$$(x^2+x+1)' = 2x+1$$

$$\Rightarrow \int \frac{x^2}{x^3-1}\,dx = \frac{1}{3}\ln|x-1| + \frac{1}{3}\ln(x^2+x+1)$$

$$+\frac{1}{3}x^3 + C$$

$$\frac{1}{3}\ln\Big(|x-1|\cdot\underbrace{(x^2+x+1)}_{>0}\Big) =$$

$$\frac{1}{3}\ln\left|(x-1)(x^2+x+1)\right| =$$

$$\frac{1}{3}\ln|x^3-1|$$