a) $\int \sin^2 x \cdot \cos^2 x \, dx$

$= \int \sin^2 x \cdot (1 - \sin^2 x) \, dx$

$= \int (\sin^2 x - \sin^4 x) \, dx$

$= \int \sin^2 x \, dx - \int \sin^4 x \, dx$

$$= \frac{x}{2} - \frac{\sin(2x)}{4} - \left(\frac{12x - 8\sin(2x) + \sin(4x)}{32} \right) + C$$

d'après le 2.2.6 b et d, ce qui permet de terminer le calcul.

b) $$\int \sqrt{\sin x}\ \cos^3 x\ dx =$$

$$\int (\sin x)^{\frac{1}{2}} \cdot \cos^2 x \cdot \cos x\ dx =$$

$$\int (\sin x)^{\frac{1}{2}} \cdot (1 - \sin^2 x)\ \cos x\ dx =$$

$$\int \left[(\sin x)^{\frac{1}{2}} - (\sin x)^{\frac{1}{2}+2} \right] \cos x\ dx =$$

$$\int (\underbrace{\sin x}_{})^{\frac{1}{2}} \cdot \underset{(\,)'}{\cos x}\ dx - \int (\underbrace{\sin x}_{})^{\frac{5}{2}} \underset{(\,)'}{\cos x}\ dx =$$

$$\frac{1}{\frac{1}{2}+1} (\sin x)^{\frac{1}{2}+1} - \frac{1}{\frac{5}{2}+1} \cdot (\sin x)^{\frac{5}{2}+1} + C =$$

$$\frac{2}{3} (\sin x)^{\frac{3}{2}} - \frac{2}{7} (\sin x)^{\frac{7}{2}} + C$$

c) $\cos\alpha \sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta) \right)$

$\Rightarrow \int \sin(5x) \cos(3x)\, dx =$

$\frac{1}{2}\int \left(\sin(3x+5x) - \sin(3x-5x) \right) dx =$

$\sin(-\alpha) = -\sin\alpha$

$\frac{1}{2}\int \left(\sin(8x) - \sin(-2x) \right) dx =$

$\frac{1}{2}\int \sin(8x)\, dx + \frac{1}{2}\int \sin(2x)\, dx =$

$\frac{1}{2}\cdot\frac{1}{8}\int \sin(8x)\cdot 8\cdot dx + \frac{1}{2}\cdot\frac{1}{2}\int \sin(2x)\cdot 2\cdot dx,$

ce qui permet de trouver la primitive.

d) $\int \frac{1}{2-\sin x} \cdot \cos x \, dx =$

$-1 \cdot \int \frac{1}{\boxed{\sin x - 2}} \cdot \cos x \, dx =$

$(\sin x - 2)' = \cos x - 0 = \cos x$

$-1 \cdot \ln|\sin x - 2| + C$