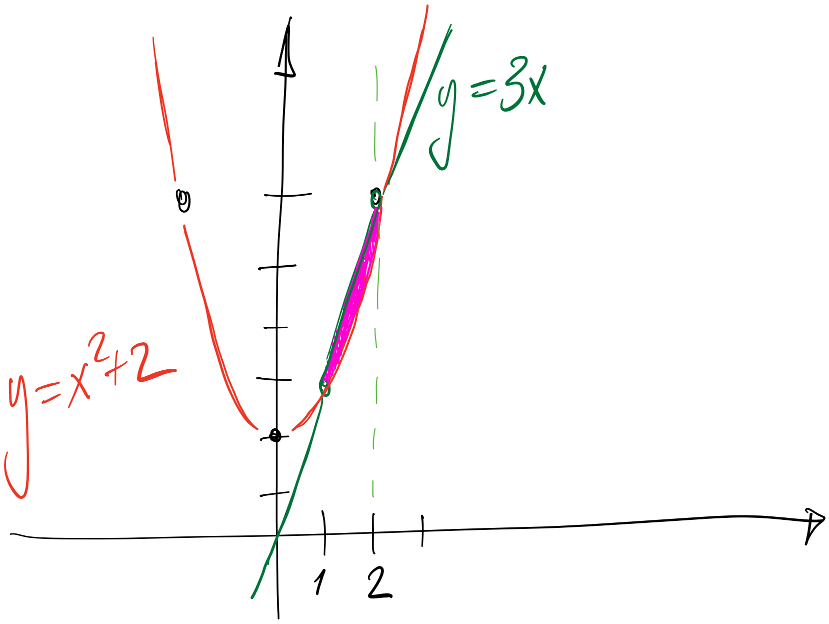
$y = x^2 + 2$

$y = 3x$



$x^2 + 2 = 3x \iff (x-1)(x-2) = 0$

$\iff x = 1 \;/\; x = 2$

a) $\pi \displaystyle\int_1^2 \left[(3x)^2 - (x^2+2)^2\right] dx$

b) $2\pi \displaystyle\int_1^2 \left[x \cdot (x^2+2) - x \cdot 3x\right] dx$

Indications

Détails des calculs : voir plus bas

c) $x = 1$ devient $x = 0$ si x est remplacé par $(x+1)$.

⚠ À changer les bornes !

BORNES : $x+1 = 1 \mid x=0$

$x+1 = 2 \mid x=1$

$$x^2+2 \longrightarrow (x+1)^2+2$$

$$3x \longrightarrow 3(x+1)$$

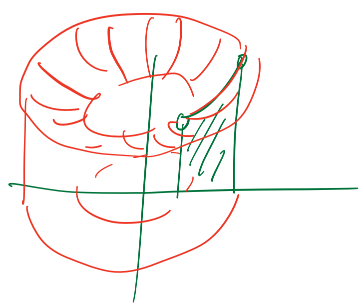
$$\Rightarrow V_1 = 2\pi \int_0^1 x \cdot \left[(x+1)^2+2\right] dx = \frac{29}{6}\pi$$

$$V_2 = 2\pi \int_0^1 x \cdot 3 \cdot (x+1)\, dx = 5\pi$$

$$\left| V_1 - V_2 \right| = \frac{\pi}{6}$$

b) Détails des calculs

$$2\pi \int_a^b x\, f(x)\, dx$$



$$2\pi \cdot \int_1^2 x \cdot (x^2+2) \cdot dx$$

$$2\pi \cdot \int_1^2 x \cdot 3 \cdot x\, dx$$

Calculer la différence

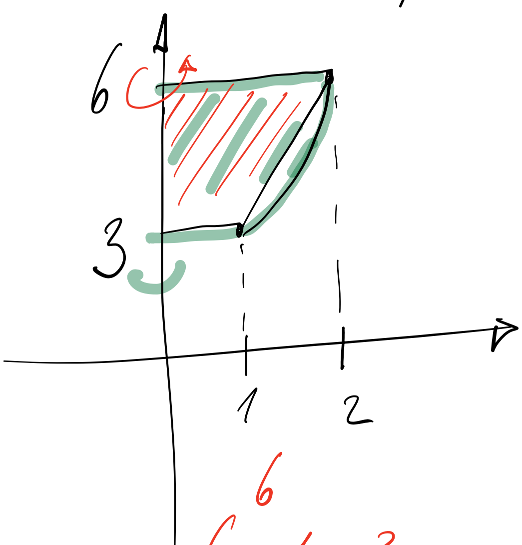
$$\int_1^2 (x^3+2x-3x^2)\,dx =$$

$$\frac{1}{4}x^4 - x^3 + x^2 \Big|_1^2 = (4-8+4) - \left(\frac{1}{4} - 1 + 1\right)$$

$$= -\frac{1}{4}$$

$$\Rightarrow V = 2\pi \cdot \left|-\frac{1}{4}\right| = \frac{\pi}{2}$$

Autre méthode pour le b) :



$y = 3x \qquad x \in [1;2]$

$x = \frac{1}{3}y \qquad y \in [3;6]$

$$V = \pi \int_3^6 \left(\frac{1}{3}y\right)^2 dy = \frac{\pi}{9} \cdot \frac{1}{3}y^3 \Big|_3^6 = \frac{\pi}{27} \cdot (216 - 27)$$

$$= \frac{\pi}{27} \cdot 189 = 7\pi$$

$$y = x^2 + 2 \quad x \in [1; 2]$$

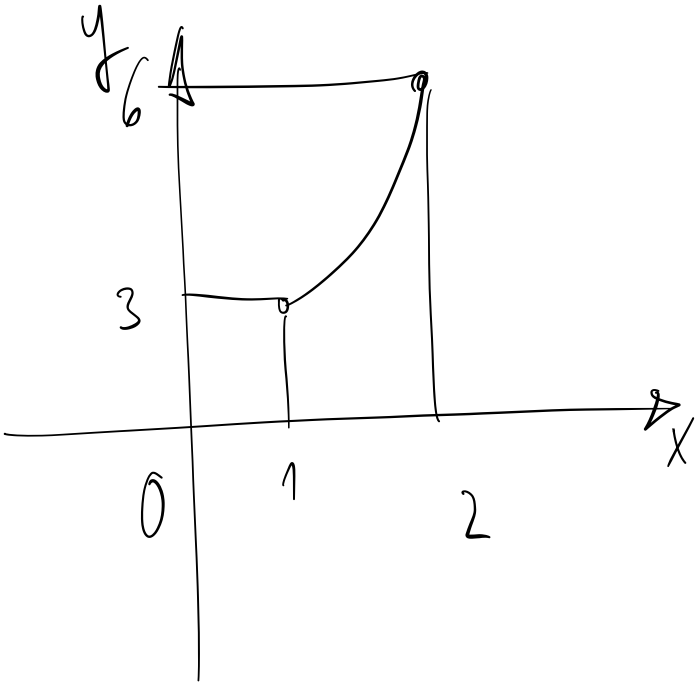
$$x = \sqrt{y-2} \quad x \in [3; 6]$$

$$V = \pi \int_3^6 \left(\sqrt{y-2}\right)^2 dy$$

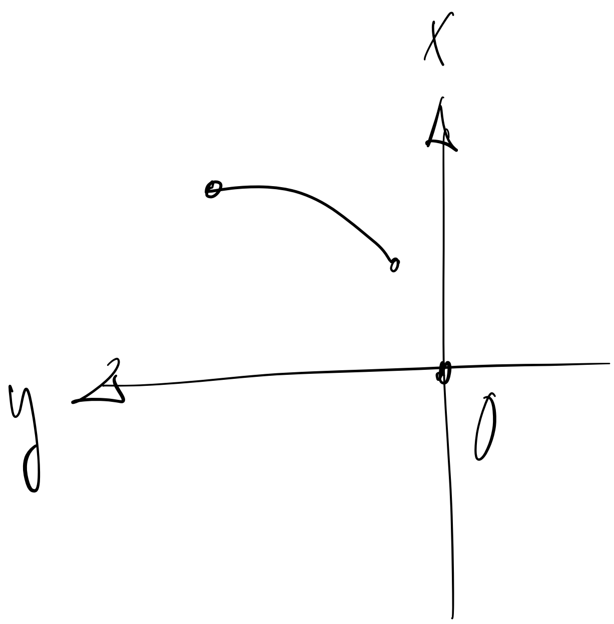
$$= \pi \int_3^6 y - 2 = \pi \left(\frac{1}{2} y^2 - 2y\right)\Big|_3^6$$

$$= \pi \cdot \left[18 - 12 - (4{,}5 - 6)\right]$$

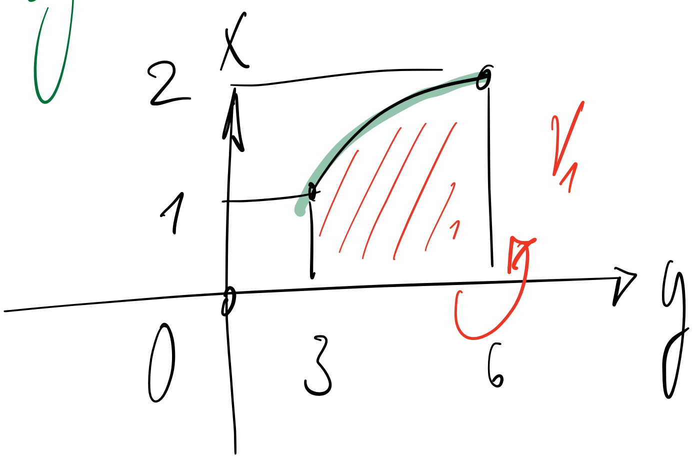
$$= \pi \cdot 7{,}5 \quad \Rightarrow \quad V - V = \frac{\pi}{2}$$



1/4 tour



Symétrie d'axe Ox



$$y = x^2 + 2 \Leftrightarrow y - 2 = x^2 \Leftrightarrow \boxed{x = \sqrt{y-2}}$$

$$V_2 = \pi \int_3^6 \left(\sqrt{y-2}\right)^2 dy$$

d) $x=2 \longrightarrow x=0$ ← AXES

$\boxed{x \longrightarrow x+2}$

$1 \longrightarrow -1$

$2 \longrightarrow 0$

On doit donc calculer

$$V_1 = 2\pi \int_{-1}^{0} x \cdot 3 \cdot (x+2)\, dx = -4\pi$$

$$V_2 = 2\pi \int_{-1}^{0} x\left((x+2)^2+2\right) dx = -\frac{23\pi}{6}$$

On en déduit $V = |V_1 - V_2| = \frac{\pi}{6}$

e) Pour « faire tourner » $f(x)$ autour de $y=3$, il suffit de le faire pour $f(x)-3$ autour de $y=0$.

axe Ox

$$V_1 = \pi \int_1^2 (3x-3)^2\,dx = 3\pi$$

$$V_2 = \pi \int_1^2 (x^2+2-3)^2\,dx = \frac{38\pi}{15}$$

$$V = |V_1 - V_2| = \frac{7\pi}{15}$$