a) $t = x+6 \iff x = t-6$ $\quad x=1 \Rightarrow t=7$

$\boxed{dx = dt}$ $\quad x=2 \Rightarrow t=8$

$$\int_1^2 \frac{x}{x+6}\,dx = \int_7^8 \frac{t-6}{t}\,dt$$

$$= \int_7^8 \left(1-\frac{6}{t}\right)dt = \int_7^8 1\,dt - 6\int_7^8 \frac{1}{t}\,dt$$

$$= t\Big|_7^8 - 6\ln|t|\Big|_7^8 = (8-7) - 6(\ln 8 - \ln 7)$$

$$= 1 - 6\ln\frac{8}{7} = 1 + 6\ln\frac{7}{8}$$

b) $$\int_0^4 \sqrt{x}(x+2)\,dx = \int_0^4 \left(x^{\frac{1}{2}}\cdot x + 2x^{\frac{1}{2}}\right)dx$$

$$= \int_0^4 x^{\frac{3}{2}}\,dx + 2\int_0^4 x^{\frac{1}{2}}\,dx$$

$$= \frac{1}{\frac{3}{2}+1}\cdot x^{\frac{3}{2}+1}\Big|_0^4 + 2\,\frac{1}{\frac{1}{2}+1}\,x^{\frac{1}{2}+1}\Big|_0^4$$

$$= \frac{2}{5}\,x^{\frac{5}{2}}\Big|_0^4 + 2\cdot\frac{2}{3}\cdot x^{\frac{3}{2}}\Big|_0^4$$

$$= \frac{2}{5}\cdot 2^5 + \frac{4}{3}\cdot 2^3 = \frac{64}{5} + \frac{32}{3} = \frac{192+160}{15}$$

$$= \frac{352}{15}$$

c) $$\int_0^{\pi/2} (\underbrace{\sin x}_{(\;)'})^5 \cos x\,dx = \frac{1}{6}(\sin x)^6\Big|_0^{\pi/2}$$

$$= \frac{1}{6}\left[\left(\sin\frac{\pi}{2}\right)^6 - (\sin 0)^6\right] = \frac{1}{6}(1-0) = \frac{1}{6}$$

f) $\displaystyle\int_2^3 \frac{5x-2}{x(x-1)}\,dx$

On observe que $\dfrac{3}{x-1} + \dfrac{2}{x} = \dfrac{5x-2}{x(x-1)}$

et donc, $\displaystyle\int_2^3 \frac{5x-2}{x(x-1)}\,dx = \int_2^3 \frac{3}{x-1}\,dx + \int_2^3 \frac{2}{x}\,dx$

$$= 3\ln|x-1|\Big|_2^3 + 2\ln|x|\Big|_2^3$$

$$= 3\ln 2 + 2\ln 3 - 2\ln 2$$

$$= \ln 2 + 2\ln 3 = \ln 2 + \ln 3^2 = \ln(2\cdot 3^2)$$

$$= \ln 18$$

g) $\displaystyle\int_{-1}^{0} \frac{1}{(x+1)^2+1}\,dx$

$t = x+1$ $\quad$ $x=-1 \Rightarrow t=0$

$x = t-1$ $\quad$ $x=0 \Rightarrow t=1$

$dx = dt$

$$= \int_0^1 \frac{1}{t^2+1}\,dt = \arctan(t)\Big|_0^1 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$