a) $x = t^2 - 2 \Leftrightarrow \sqrt{x+2} = t \qquad x \in [-2;0]$

$dx = 2t\,dt \qquad x = -2 \Rightarrow t = 0 \,/\, x = 0 \Rightarrow t = \sqrt{2}$

$$\int_{-2}^{0} x\sqrt{x+2}\,dx = \int_{0}^{\sqrt{2}} (t^2-2)\cdot t \cdot 2t\,dt$$

$$= \int_{0}^{\sqrt{2}} (2t^4 - 4t^2)\,dt = \left(\frac{2}{5}t^5 - \frac{4}{3}t^3\right)\Big|_{0}^{\sqrt{2}}$$

$$= \frac{2}{5}\sqrt{2^5} - \frac{4}{3}\sqrt{2^3} = \frac{8\sqrt{2}}{5} - \frac{8\sqrt{2}}{3}$$

$$= \frac{24-40}{15}\sqrt{2} = -\frac{16\sqrt{2}}{15}$$

b) $x = 3\sin(t) \Leftrightarrow t = \arcsin\left(\frac{x}{3}\right)$

Si $x \in [0;3]$, $t \in [0; \frac{\pi}{2}]$, ce qui est une bonne chose. On a $dx = 3\cos(t)\,dt$

$x=0 \Rightarrow t=0 \;/\; x=3 \Rightarrow t=\frac{\pi}{2}$

$$\int_0^3 \sqrt{9-x^2}\,dx = \int_0^{\pi/2} \sqrt{9-9\sin^2(t)}\;3\cos(t)\,dt$$

$$= 9\int_0^{\pi/2} \sqrt{1-\sin^2(t)}\;\cos(t)\,dt = 9\int_0^{\pi/2} \sqrt{\cos^2(t)}\,\cos(t)\,dt$$

$$= 9\int_0^{\pi/2} \cos^2(t)\,dt = 9\cdot\left(\frac{t}{2}+\frac{\sin(2t)}{4}\right)\Bigg|_0^{\pi/2}$$

$$= 9\cdot\left(\frac{\pi}{4}+\underbrace{\frac{\sin\pi}{4}-\frac{0}{2}-\frac{\sin(0)}{4}}_{0}\right)$$

$$= \frac{9\pi}{4}$$

c) $\displaystyle\int_2^4 \frac{1}{x\sqrt{x-1}}\,dx$

$x = u^2+1$

$dx = 2u \cdot du$

$x=2 \Rightarrow u=1$

$x=4 \Rightarrow u=\sqrt{3}$

$$= \int_1^{\sqrt{3}} \frac{1}{(u^2+1)\cdot \cancel{u}}\, 2\cancel{u} \cdot du$$

$$= \int_1^{\sqrt{3}} \frac{2}{u^2+1}\, du = 2\int_1^{\sqrt{3}} \frac{1}{u^2+1}\, du$$

$$= 2 \arctan(u) \Big|_1^{\sqrt{3}}$$

$$= 2\left(\arctan(\sqrt{3}) - \arctan(1)\right) = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \frac{\pi}{6}$$