

$$\begin{array}{ccc}
 \mathbb{R}_B^2 & \xrightarrow{H} & \mathbb{R}_B^2 \\
 \mathcal{P} \uparrow & & \downarrow \mathcal{P}^{-1} \\
 \mathbb{R}_{B'}^2 & \xrightarrow{H'} & \mathbb{R}_{B'}^2
 \end{array}
 \quad
 \begin{array}{l}
 \mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \\
 \mathcal{B}' = \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix} \right)
 \end{array}$$

D'après le diagramme, on peut écrire

$$H' = \mathcal{P}^{-1} H \mathcal{P}$$

$$\mathcal{P} = \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix} \quad \text{Calculons } \mathcal{P}^{-1} :$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array} \right), \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 3 & -1 & 1 \end{array} \right), \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -1/3 & 1/3 \end{array} \right)$$

$$\Rightarrow H' = \begin{pmatrix} 1 & 0 \\ -1/3 & 1/3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -1/3 & 1/3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

$$b) \mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \quad \mathcal{B}' = \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right)$$

$$\begin{array}{ccc} \mathbb{R}_{\mathcal{B}}^2 & \xrightarrow{H} & \mathbb{R}_{\mathcal{B}}^2 & H = \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix} \\ \uparrow \underline{P} & & \downarrow \underline{P}^{-1} & \\ \mathbb{R}_{\mathcal{B}'}^2 & \xrightarrow{H'} & \mathbb{R}_{\mathcal{B}'}^2 & \underline{P} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \end{array}$$

Calcul de \underline{P}^{-1} : $\left(\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right)$

$$\left(\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right), \left(\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 0 & 3/2 & -1/2 & 1 \end{array} \right)$$

$$\left(\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/6 & 1/3 \end{array} \right), \left(\begin{array}{cc|cc} 1 & 0 & 1/3 & 1/3 \\ 0 & 1 & -1/3 & 2/3 \end{array} \right)$$

$$H' = \underline{P}^{-1} H \underline{P} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 5 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 4 \\ -4 & 11 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 & 4/3 \\ -4/3 & 11/3 \end{pmatrix}$$