2) $\mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)$

$\mathcal{B}' = \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right)$

$$\begin{array}{ccc} \mathbb{R}^3_{\mathcal{B}} & \xrightarrow{H} & \mathbb{R}^3_{\mathcal{B}} \\ {\scriptstyle P}\uparrow & & \downarrow{\scriptstyle P^{-1}} \\ \mathbb{R}^3_{\mathcal{B}'} & \xrightarrow{H'} & \mathbb{R}^3_{\mathcal{B}'} \end{array}$$

$H = \begin{pmatrix} 2 & 3 & 1 \\ 1 & -2 & 0 \\ -1 & 4 & 0 \end{pmatrix}$

$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

On constate que $P \cdot P = I_3$, $P = P^{-1}$

$\Rightarrow H' = P^{-1} \cdot H \cdot P = PHP$

$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 \\ 1 & -2 & 0 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 4 & -1 \\ 0 & -2 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

b) $B' = \left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right)$

Dans ce cas, $P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

et $P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

$$\Rightarrow H' = P^{-1} H P$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 \\ 1 & -2 & 0 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 3 & -1 \\ -4 & -4 & 2 \\ 7 & 6 & 1 \end{pmatrix}$$