$m = f \circ h$

$$\Rightarrow M = F \cdot H = \begin{pmatrix} -1 & 2 & 0 \\ 2 & -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -2 & 1 \\ 0 & -1 \end{pmatrix}$$

$$M = \begin{pmatrix} -5 & 5 \\ 8 & -8 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\operatorname{Im}(m) = \left\langle \begin{pmatrix} -5 \\ 8 \end{pmatrix} \right\rangle$$

$$\ker(m): \quad \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow x = y \quad \Rightarrow \ker(m) = \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\rangle$$

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$p = g \circ i$

$\Rightarrow P = G \cdot I$

$$P = G \cdot \underline{V} = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -6 \\ 1 & -3 \end{pmatrix}$$

$$\Rightarrow \quad P = \begin{pmatrix} -1 & 3 \\ 5 & -15 \end{pmatrix}$$

$$\operatorname{Im}(p) = \left\langle \begin{pmatrix} -1 \\ 5 \end{pmatrix}; \begin{pmatrix} 3 \\ -15 \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{pmatrix} -1 \\ 5 \end{pmatrix} \right\rangle$$

$$\ker(p): \quad \begin{pmatrix} -1 & 3 \\ 5 & -15 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} x = 3y \\ y = y \end{cases} \Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = k \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\Rightarrow \ker(p) = \left\langle \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\rangle$$

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$$q = (f+g) \circ h$$

$$\Rightarrow Q = (F+G) \cdot H$$

$$Q = \begin{pmatrix} 0 & 1 & 2 \\ 0 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -2 & 1 \\ 0 & -1 \end{pmatrix}$$

$$Q = \begin{pmatrix} -2 & -1 \\ 4 & -2 \end{pmatrix}$$

$$\mathrm{Im}(g) = \left\langle \begin{pmatrix} -2 \\ 4 \end{pmatrix}; \begin{pmatrix} -1 \\ -2 \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{pmatrix} 1 \\ -2 \end{pmatrix}; \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{pmatrix} 2 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\rangle = \mathbb{R}^2$$

$\Rightarrow$ $\ker(g) = \{0\}$ par le théorème du rang.