2MR

$\boxed{2.6.5}$

a) $\lim\limits_{x \to 0^{>}} \frac{|x|}{x} = \lim\limits_{x \to 0^{>}} \frac{x}{x} = 1$

$\lim\limits_{x \to 0^{<}} \frac{|x|}{x} = \lim\limits_{x \to 0^{<}} \frac{-x}{x} = -1$

$\lim\limits_{x \to 0} \frac{|x|}{x}$ n'existe pas

b) $\lim\limits_{x \to 0^{>}} \frac{x^2 + |x|}{|x|} = \lim\limits_{x \to 0} \frac{x^2 + x}{x}$

$= \lim\limits_{x \to 0} \frac{x(x+1)}{x}$

$= \lim\limits_{x \to 0} x + 1 = 1$

$\boxed{2.6.5}_2$ 2MR

b) $$\lim_{\substack{x\to 0\\ <}} \frac{x^2+|x|}{|x|} = \lim_{\substack{x\to 0\\ <}} \frac{x^2-x}{-x}$$

$$= \lim_{\substack{x\to 0\\ <}} \frac{x(x-1)}{x(-1)} = \lim_{\substack{x\to 0\\ <}} -x+1$$

$$= 1$$

Finalement, $$\lim_{x\to 0} \frac{x^2+|x|}{|x|} = 1$$

c) $$\lim_{\substack{x\to 0\\ >}} \frac{x^2-2x}{|x|} = \lim_{\substack{x\to 0\\ >}} x-2 = -2$$

$$\lim_{\substack{x\to 0\\ <}} \frac{x^2-2x}{|x|} = \lim_{\substack{x\to 0\\ <}} -x+2 = 2$$

$$\Rightarrow \lim_{x\to 0} \frac{x^2-2x}{|x|}$$ n'existe pas

$\boxed{2.6.5}_3$ 2MR

d) $$\lim_{\substack{x \to 2 \\ >}} \frac{|x-2|}{x^2-3x+2} = \lim_{\substack{x \to 2 \\ >}} \frac{x-2}{(x-2)(x-1)}$$

$$= \lim_{\substack{x \to 2 \\ >}} \frac{1}{x-1} = 1$$

$$\lim_{\substack{x \to 2 \\ <}} \frac{|x-2|}{x^2-3x+2} = \lim_{\substack{x \to 2 \\ <}} \frac{2-x}{(x-2)(x-1)}$$

$$= \lim_{\substack{x \to 2 \\ <}} \frac{-1}{x-1} = -1$$

$$\Rightarrow \lim_{x \to 2} \frac{|x-2|}{x^2-3x+2}$$ n'existe pas

$\boxed{2.6.6}$ 2MR

On sait que $\sin(a) \in [-1;1]$

$\forall a \in \mathbb{R}$

$$\Rightarrow \quad -1 \leq \sin\left(\frac{1}{x}\right) \leq 1 \quad \forall x \neq 0$$

$$\Leftrightarrow \quad -x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

$$\Rightarrow \quad -\lim_{x \to 0} x^2 \leq \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \leq \lim_{x \to 0} x^2$$

$$\Rightarrow \quad 0 \leq \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \leq 0$$

$$\Rightarrow \quad \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

$\boxed{2.6.6}_2$ 2MR

On sait que $\cos(a) \in [-1;1] \ \forall a \in \mathbb{R}$

$$\Rightarrow \quad -1 \leq \cos\left(\frac{1}{x}\right) \leq 1 \quad \forall x \neq 0$$

$$\Leftrightarrow \quad -\sqrt{x} \leq \sqrt{x} \cdot \cos\left(\frac{1}{x}\right) \leq \sqrt{x}$$

$$\Rightarrow \quad -\lim_{x \underset{>}{\to} 0} \sqrt{x} \leq \lim_{x \underset{>}{\to} 0} \sqrt{x} \cdot \cos\left(\frac{1}{x}\right) \leq \lim_{x \underset{>}{\to} 0} \sqrt{x}$$

$$\Rightarrow \quad 0 \leq \lim_{x \underset{>}{\to} 0} \sqrt{x} \cdot \cos\left(\frac{1}{x}\right) \leq 0$$

$$\Rightarrow \quad \lim_{x \underset{>}{\to} 0} \sqrt{x} \cdot \cos\left(\frac{1}{x}\right) = 0$$

$\boxed{2.6.7}$ 2MR

On calcule d'abord $\lim\limits_{\substack{x \to 0 \\ >}} (x \cdot f(x))$:

$$0 \leq f(x) \leq 3 \qquad \forall x > 0$$

$$\Rightarrow \quad 0 \leq x\, f(x) \leq 3x \quad \text{vu que } x > 0$$

$$\Rightarrow \quad 0 \leq \lim_{\substack{x \to 0 \\ >}} x \cdot f(x) \leq \lim_{\substack{x \to 0 \\ >}} 3x$$

$$\Rightarrow \quad 0 \leq \lim_{\substack{x \to 0 \\ >}} x \cdot f(x) \leq 0$$

$$\Rightarrow \quad \lim_{\substack{x \to 0 \\ >}} x\, f(x) = 0$$

$\boxed{2.6.7}_2$ 2MR

On calcule ensuite $\lim\limits_{\substack{x \to 0 \\ <}} (x \cdot f(x))$:

$$0 \leq f(x) \leq 3 \qquad \forall x < 0$$

$\Rightarrow \quad 0 \geqslant x f(x) \geqslant 3x$ vu que $x < 0$

$$\Rightarrow \quad 0 \geqslant \lim_{\substack{x \to 0 \\ <}} x \cdot f(x) \geqslant \lim_{\substack{x \to 0 \\ <}} 3x$$

$$\Rightarrow \quad 0 \geqslant \lim_{\substack{x \to 0 \\ <}} x f(x) \geqslant 0$$

$$\Rightarrow \quad \lim_{\substack{x \to 0 \\ <}} x f(x) = 0$$

En fin de compte, $\lim\limits_{x \to 0} x f(x) = 0$

$$\boxed{2.6.8}$$

2MR

a) $$\lim_{x\to 0} \frac{\sin(2x)}{x} = \lim_{x\to 0} \frac{2}{2}\cdot\frac{\sin(2x)}{x}$$

$t = 2x \Rightarrow (x\to 0 \Leftrightarrow t\to 0)$

$$= \lim_{x\to 0} 2\cdot\frac{\sin(2x)}{2x} = \lim_{t\to 0} 2\cdot\frac{\sin(t)}{t}$$

$$= 2\cdot\lim_{t\to 0}\frac{\sin(t)}{t} = 2\cdot 1 = 2$$

b) $$\lim_{x\to 0}\frac{\sin(3x)}{\sin(2x)}$$

$$= \lim_{x\to 0}\frac{\sin(3x)}{3x}\cdot\frac{2x}{\sin(2x)}\cdot\frac{3}{2}$$

$$= \lim_{x\to 0}\frac{\sin(3x)}{3x}\cdot\lim_{x\to 0}\frac{1}{\frac{\sin(2x)}{2x}}\cdot\lim_{x\to 0}\frac{3}{2}$$

$\boxed{2.6.8}_2$ 2MR

b) (suite) $\Rightarrow \lim\limits_{x \to 0} \dfrac{\sin(3x)}{\sin(2x)} =$

$$\lim_{t \to 0} \frac{\sin(t)}{t} \cdot \frac{1}{\lim\limits_{y \to 0} \dfrac{\sin(y)}{y}} \cdot \frac{3}{2} =$$

$$1 \cdot \frac{1}{1} \cdot \frac{3}{2} = \frac{3}{2}$$

c) $\lim\limits_{x \to 0} \dfrac{\sin(x/4)}{5x} = \lim\limits_{x \to 0} \dfrac{1}{5} \cdot \dfrac{\sin(x/4)}{4 \cdot (x/4)}$

$= \dfrac{1}{20} \cdot \lim\limits_{x \to 0} \dfrac{\sin(x/4)}{x/4}$ $\left(t = \dfrac{x}{4} \Rightarrow (x \to 0 \Leftrightarrow t \to 0) \right)$

$= \dfrac{1}{20} \cdot \lim\limits_{t \to 0} \dfrac{\sin(t)}{t} = \dfrac{1}{20}$

2MR

$\boxed{2.6.8}_3$

d) $$\lim_{x\to 0}\frac{\tan(7x)}{\sin(3x)} = \lim_{x\to 0}\frac{\frac{\sin(7x)}{\cos(7x)}}{\sin(3x)} =$$

$$\lim_{x\to 0}\frac{\sin(7x)}{7x}\cdot\frac{3x}{\sin(3x)}\cdot\frac{1}{\cos(7x)}\cdot\frac{7}{3} =$$

$$\lim_{x\to 0}\frac{\sin(7x)}{7x}\cdot\lim_{x\to 0}\frac{3x}{\sin(3x)}\cdot\lim_{x\to 0}\frac{1}{\cos(7x)}\cdot\frac{7}{3} =$$

$$\lim_{x\to 0}\frac{\sin(7x)}{7x}\cdot\frac{1}{\lim_{x\to 0}\frac{\sin(3x)}{3x}}\cdot\lim_{x\to 0}\frac{1}{\cos(7x)}\cdot\frac{7}{3} =$$

$y = 7x \quad (x\to 0 \Leftrightarrow y\to 0)$

$t = 3x \quad (x\to 0 \Leftrightarrow t\to 0)$

$$\Rightarrow \lim_{x\to 0}\frac{\tan(7x)}{\sin(3x)} = \lim_{y\to 0}\frac{\sin(y)}{y}\cdot\frac{1}{\lim_{t\to 0}\frac{\sin(t)}{t}}\cdot 1\cdot\frac{7}{3}$$

$$= \frac{7}{3}$$

$\boxed{2.6.8}_4$ 2MR

e) $\lim\limits_{x \to 1} \frac{\sin(x-1)}{x-1} = \lim\limits_{t \to 0} \frac{\sin(t)}{t} = 1$

$x-1 = t : \quad x \to 1 \Leftrightarrow t \to 0$

f) $\lim\limits_{x \to 0} \frac{\tan(x)}{x} = \lim\limits_{x \to 0} \frac{\frac{\sin(x)}{\cos(x)}}{x}$

$= \lim\limits_{x \to 0} \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{x}$

$= \lim\limits_{x \to 0} \frac{\sin(x)}{x} \cdot \frac{1}{\cos(x)}$

$= \lim\limits_{x \to 0} \frac{\sin(x)}{x} \cdot \lim\limits_{x \to 0} \frac{1}{\cos(x)}$

$= 1 \cdot 1 = 1$

$$\boxed{2.6.8}$$

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2MR

$$\cos^2(x) + \sin^2(x) = 1$$

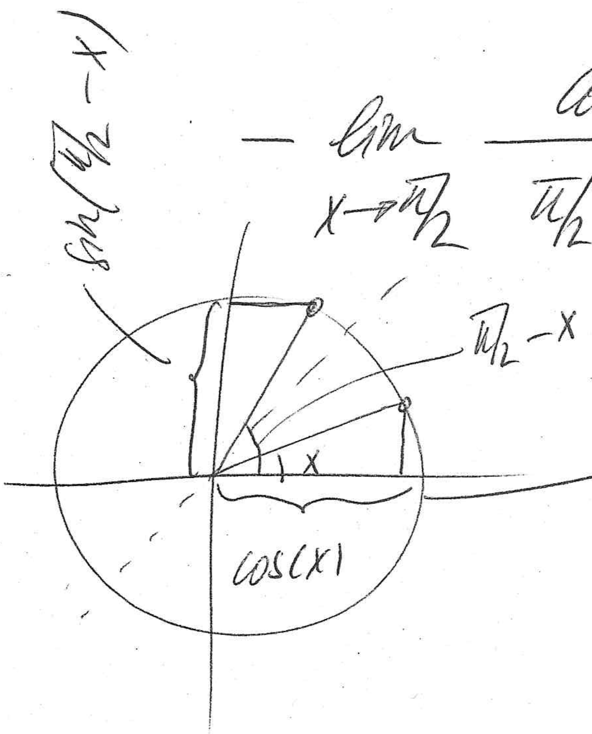
g) $$\lim_{x \to 0} \frac{1 - \cos(x)}{\sin^2(x)} = \lim_{x \to 0} \frac{1 - \cos(x)}{1 - \cos^2(x)} =$$

$$\lim_{x \to 0} \frac{\cancel{(1 - \cos(x))} \cdot 1}{\cancel{(1 - \cos(x))}(1 + \cos(x))} = \lim_{x \to 0} \frac{1}{1 + \cos(x)}$$

$$= \frac{1}{2}$$

h) $$\lim_{x \to \pi/2} \frac{\cos(x)}{x - \pi/2} = \lim_{x \to \pi/2} \frac{\cos(x)}{-(\pi/2 - x)} =$$

$$-\lim_{x \to \pi/2} \frac{\cos(x)}{\pi/2 - x} = -\lim_{x \to \pi/2} \frac{\sin(\pi/2 - x)}{\pi/2 - x}$$



$$t = \pi/2 - x : \quad x \to \pi/2 \Leftrightarrow t \to 0$$

$$= -\lim_{t \to 0} \frac{\sin(t)}{t}$$

$$= -1$$

$$\boxed{2.6.9}$$

2MR

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x} \cdot \frac{\cos(x) + 1}{\cos(x) + 1} =$$

$$\lim_{x \to 0} \frac{\cos^2(x) - 1}{x \cdot (\cos(x) + 1)} = \lim_{x \to 0} \frac{-(1 - \cos^2(x))}{x \cdot (\cos(x) + 1)}$$

$\cos^2(x) + \sin^2(x) = 1$

$$= \lim_{x \to 0} \frac{-\sin^2(x)}{x(\cos(x) + 1)} = -\lim_{x \to 0} \frac{\sin(x)}{x} \cdot \frac{\sin(x)}{\cos(x)+1}$$

$$= (-1) \cdot \lim_{x \to 0} \frac{\sin(x)}{x} \cdot \lim_{x \to 0} \frac{\sin(x)}{\cos(x)+1}$$

$$= -1 \cdot 1 \cdot \left\langle\!\left\langle \frac{0}{1+1} \right\rangle\!\right\rangle = -1 \cdot 1 \cdot 0$$

$$= 0$$

$\boxed{2.6.9}_2$ 2MR

$$\lim_{x \to 0} \frac{1-\cos(x)}{x^2} \cdot \frac{\cos(x)+1}{\cos(x)+1} =$$

$\cos^2(x)+\sin^2(x)=1$

$$\lim_{x \to 0} \frac{1-\cos^2(x)}{x^2 \cdot (\cos(x)+1)} = \lim_{x \to 0} \frac{\sin^2(x)}{x^2(\cos(x)+1)} =$$

$$\lim_{x \to 0} \frac{\sin^2(x)}{x^2} \cdot \lim_{x \to 0} \frac{1}{\cos(x)+1} =$$

$$\lim_{x \to 0} \left(\frac{\sin(x)}{x}\right)^2 \cdot \lim_{x \to 0} \frac{1}{\cos(x)+1} =$$

$$\left[\lim_{x \to 0} \frac{\sin(x)}{x}\right]^2 \cdot \lim_{x \to 0} \frac{1}{\cos(x)+1} = 1 \cdot \frac{1}{2}$$

$$= \frac{1}{2}$$