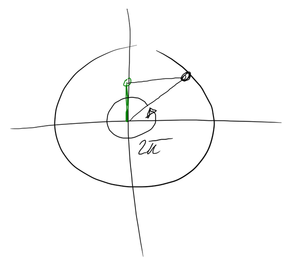
$\boxed{2.10.10}$ j) $\sin^2(x) - 2\cos(x) = f(x)$

$D_f = \mathbb{R}$

PÉRIODE

	période
sin	2π
cos	2π



INTERVALLE D'ÉTUDE : $[0; 2\pi]$

Zéro

$\sin^2 x - 2\cos x = 0$ $\qquad \cos^2 x + \sin^2 = 1$

$1 - \cos^2 x - 2\cos x = 0$

$\cos^2 x + 2\cos x - 1 = 0$ $\qquad \frac{-2 \pm \sqrt{8}}{2}$

$\cos x = \frac{-2 \pm \sqrt{4 - 4\cdot 1\cdot(-1)}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$ $\quad 0{,}4142$; $-2{,}4142$

$\cos x = 0{,}4142$ $\qquad x = \pm\arccos(0{,}4142) + k2\pi$ $\qquad x = \underbrace{\pm 1{,}144}_{\text{rad}} + k2\pi$

$\cos x = \cancel{-2{,}4142}$ $\qquad x = \pm\arccos\cancel{(-2{,}4142)} + k2\pi$

$\ldots \; | x = -5{,}14 \; | x = 1{,}14 \; | x = 7{,}42 \; | \ldots$

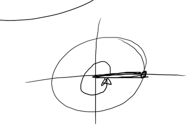
$\ldots \; | -7{,}42 \; | x = -1{,}14 \; | x = 5{,}14 \; | \ldots$

$x \in [0; 2\pi] \Leftrightarrow x \simeq 1{,}144$; $x \simeq 5{,}14$

Signe :

	0		1,14		5,14		2π	
	-2	−	0	+	0	−	-2	

$\sin^2 x - 2\cos x$



Asymptotes : Niet

Dérivée et croissance :

$f'(x) = 2\cdot\sin x\cdot(\sin x)' - 2\cdot(\cos x)' = 2\sin x\cos x + 2\sin x$

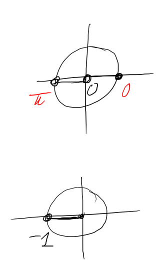
$f'(x) = 0 \Leftrightarrow 2\sin x\cos x + 2\sin x = 0$

$2\sin x(\cos x + 1) = 0$

$\sin x = 0$ $\qquad x = \begin{matrix}\pi\\0\end{matrix} + k2\pi$

ou

$\cos x = -1$ $\qquad x = \pi + k2\pi$



$-2\pi \,|\, 0 \,|\, 2\pi \,|\, 4\pi$

$\pm\pi + k2\pi$

$| -\pi \,|\, \pi \,|\, 3\pi \,|\, 5\pi$

$| -\pi \,|\, \pi \,|\, 3\pi$

$f'(x) = 0$ et $x \in [0; 2\pi] \Leftrightarrow$ $x = \pi$; $x = 0$; $x = 2\pi$

$2\sin x(\cos x + 1)$

	0		π		2π	
−	0	+	0	−	0	+
↘	min $(0;-2)$	↗	max $(\pi;2)$	↘	min $(2\pi;-2)$	↗

Dérivée seconde et courbure :

$f''(x) = 2\sin x(\cos x + 1)' = 2\cdot(\cos x(\cos x + 1) + \sin x\cdot(-\sin x))$

$= 2(\cos^2 x + \cos x - \sin^2 x)$

$= 2(\cos^2 x + \cos x - (1 - \cos^2 x))$

$= 2(2\cos^2 x + \cos x - 1)$

$\cos x = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4} < \begin{matrix}\frac{1}{2}\\ -1\end{matrix}$

$\cos x = \frac{1}{2}$ $\qquad x = \pm\arccos\left(\frac{1}{2}\right) + k2\pi = \pm\frac{\pi}{3} + k\cdot 2\pi$

$| \frac{\pi}{3} - 2\pi \,|\, \frac{\pi}{3} \,|\, 2\pi + \frac{\pi}{3} \,|\, \ldots$ $\quad 1{,}05$

$\ldots | -\frac{\pi}{3} \,|\, \underbrace{-\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}}_{5{,}2} \,|\, \ldots$

$\cos x = -1$ $\qquad x = \pi + k2\pi$ $\qquad -\pi \,|\, \pi \,|\, 3\pi$

$f''(x) = 0$ et $x \in [0; 2\pi] \Leftrightarrow$ $x = 1{,}05$; $x = \pi$; $x = 5{,}2$

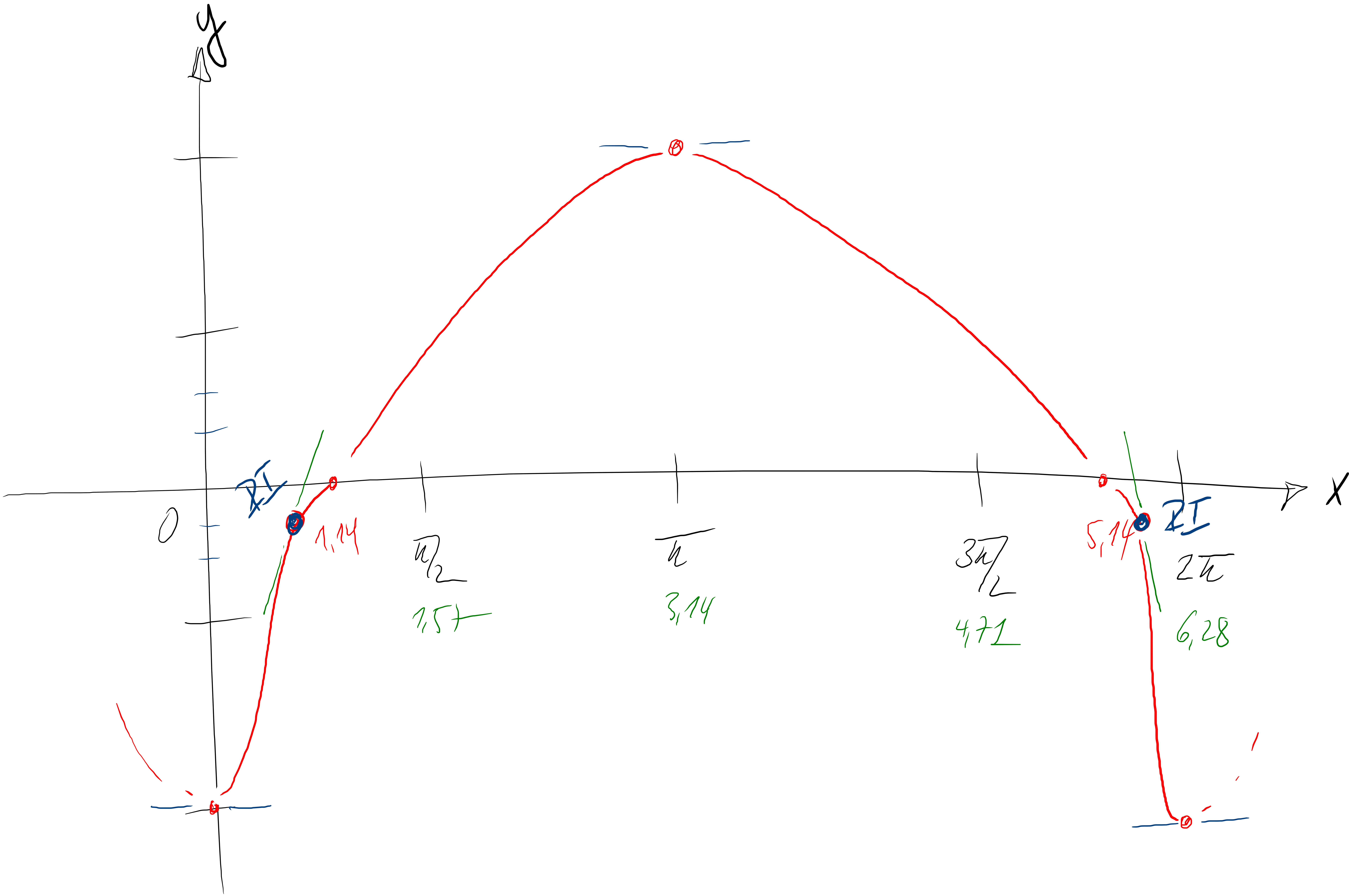
$f'(x) = 2\sin x(\cos x + 1)$

$f(x) = \sin^2 x - 2\cos x$

$2(2\cos^2 x + \cos x - 1)$

	0		1,05		3,14		5,2		2π	
+		+	0	−	0	−	0	+		+
∪	4	∪	PI	∩		∩	PI	∪	4	∪

	f'(x)	f(x)
1,05	2,6	−0,25
5,2	−2,6	−0,25



y
x
0
PI
1,14
π/2
1,57
π
3,14
3π/2
4,71
5,14
PI
2π
6,28