19.03.20

2.10.10

h) $f(x) = \sqrt{\dfrac{x^3}{x-2}}$

① Ensemble de définition

Signe de $\dfrac{x^3}{x-2}$

x		0		2	
$\dfrac{x^3}{x-2}$	$+$	0	$-$	$\parallel$	$+$

Texte

$ED(f) =]-\infty ; 0] \cup]2 ; +\infty[$

② Parité : aucune cf $ED(f)$

③ Périodicité : aucune

④ Signe de $f(x)$

x		0		2	
$f(x)$	$+$	0		$\parallel$	$+$

⑤ AV

$$\lim_{x \to 2^>} \sqrt{\frac{x^3}{x-2}} = +\infty \qquad \text{AV à droite} \quad x = 2$$

AHD : $\lim_{x \to +\infty} \sqrt{\frac{x^{\cancel{3}\,2}}{\cancel{x}(1-\frac{2}{x})}} = +\infty$ pas de AHD

AHG : $\lim_{x \to -\infty} \sqrt{\frac{x^3}{x-2}} = +\infty$ pas de AHG

AOD : $m = \lim_{x \to +\infty} \frac{\sqrt{\frac{x^3}{x-2}}}{x}$

$$= \lim_{x \to +\infty} \sqrt{\frac{x^3}{x-2}} \cdot \frac{1}{x} = \lim_{x \to +\infty} \sqrt{\frac{x^3}{x^2(x-2)}}$$

$$= \lim_{x \to +\infty} \sqrt{\frac{\cancel{x^3}}{\cancel{x^3}(1-\frac{2}{x})}} = 1$$

$$h = \lim_{x \to +\infty} \left(\sqrt{\frac{x^3}{x-2}} - x \right) = \lim_{x \to +\infty} \left(\frac{\sqrt{x^3}}{\sqrt{x-2}} - \frac{x\sqrt{x-2}}{\sqrt{x-2}} \right)$$

$$= \lim_{x \to +\infty} \frac{x\sqrt{x} - x\sqrt{x-2}}{\sqrt{x-2}} = \lim_{x \to +\infty} \frac{x(\sqrt{x} - \sqrt{x-2})}{\sqrt{x-2}} \cdot \frac{(\sqrt{x} + \sqrt{x-2})}{(\sqrt{x} + \sqrt{x-2})}$$

$$= \lim_{x \to +\infty} \frac{x(x-x+2)}{\sqrt{x}\sqrt{1-\frac{2}{x}} \cdot \sqrt{x}(1+\sqrt{1-\frac{2}{x}})} = \lim_{x \to +\infty} \frac{2\cancel{x}}{\cancel{x}\underbrace{\sqrt{1+\ldots}}_{\to 1} \cdot \underbrace{(1+\sqrt{1+\ldots})}_{\to 2}} = 1$$

$$\boxed{\text{AOD} : y = x + 1}$$

$$\boxed{\text{AOG} : y = -x - 1}$$ en exercice

Formules

$$m = \lim_{x \to \infty} \frac{f(x)}{x}$$

$$h = \lim_{x \to \infty} (f(x) - mx)$$

AH/AO à droite :

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \sqrt{\frac{x^{\cancel{3}\,2}}{x(1-\frac{2}{x})}} = \lim_{x\to+\infty} \frac{x}{\sqrt{1-\frac{2}{x}}} = +\infty \text{ pas d'AH}$$

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt{\frac{x^3}{x-2}}}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\sqrt{x}}{\cancel{x}\sqrt{x-2}} =$$

$$= \lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x\underbrace{(1-\frac{2}{x})}_{>0}}} = \lim_{x\to+\infty} \frac{\cancel{\sqrt{x}}\ 1}{\cancel{\sqrt{x}}\sqrt{1-\frac{2}{x}}} = 1 \quad AO: y = x+h$$

$$h = \lim_{x\to+\infty} \left(f(x) - x\right) = \lim_{x\to+\infty} \left(\sqrt{\frac{x^3}{x-2}} - x\right) =$$

$$\lim_{x\to+\infty} \frac{\left(\sqrt{\frac{x^3}{x-2}} - x\right)\left(\sqrt{\frac{x^3}{x-2}} + x\right)}{\sqrt{\frac{x^3}{x-2}} + x} = \lim_{x\to+\infty} \frac{\frac{x^3}{x-2} - x^2}{\frac{x\sqrt{x}}{\sqrt{x-2}} + x}$$

$$\lim_{x\to+\infty} \frac{\frac{x^3}{x-2} - x^2}{\frac{x\sqrt{x}}{\sqrt{x-2}} + x} = \lim_{x\to+\infty} \frac{x^3 - x^3 + 2x^2}{(x-2)\left[\frac{x\sqrt{x} + x\sqrt{x-2}}{\sqrt{x-2}}\right]} =$$

$$\lim_{x\to+\infty} \frac{2x^{\cancel{2}}}{\sqrt{x-2}\,\cancel{x}\,(\sqrt{x}+\sqrt{x-2})} = \lim_{x\to+\infty} \frac{2\cancel{x}}{\cancel{\sqrt{x}}\underbrace{\left(\sqrt{1-\frac{2}{x}}\right)}_{1}\cdot\cancel{\sqrt{x}}\underbrace{\left(1+\sqrt{1-\frac{2}{x}}\right)}_{2}} = 1$$

$$\Rightarrow \text{AOD } \ y = x+1$$

⑥ Croissance

$$f'(x) = \left(\sqrt{\frac{x^3}{x-2}}\right)' \qquad (\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$\left(\frac{x^3}{x-2}\right)' = \frac{3x^2(x-2) - x^3\cdot 1}{(x-2)^2} = \frac{x^2\left(3(x-2)-x\right)}{(x-2)^2}$$

$$= \frac{x^2(2x-6)}{(x-2)^2} = \frac{2x^2(x-3)}{(x-2)^2} \qquad \left\{ \frac{1}{\sqrt{\frac{a}{b}}} = \sqrt{\frac{b}{a}} \right.$$

$$f'(x) = \frac{\cancel{2}x^2(x-3)}{(x-2)^2}\cdot\frac{1}{\cancel{2}\sqrt{\frac{x^3}{x-2}}} = \boxed{\frac{x^2(x-3)}{(x-2)^2}\sqrt{\frac{x-2}{x^3}}}$$

$$\boxed{\sqrt{x^3} = x\sqrt{x}}$$

$$= \frac{x^2(x-3)}{|x|}\sqrt{\frac{\cancel{x-2}^{\,1}}{x(x-2)^{\cancel{4}\,3}}} = \frac{|x|(x-3)}{\sqrt{x(x-2)^3}} \qquad ED(f') = ED(f) - \{0\}$$

Tableau de la croissance

x		0		2		3	
$f'(x)$	$-$	‖	//////	‖	$-$	0	$+$
$f(x)$	↘	0	//////	‖	↘	min	↗

" $\dfrac{\sqrt{x}(x-3)}{(x-2)\sqrt{x-2}}$

mais $\lim\limits_{x\to 0} f'(x) = 0$

Min $(3 ; 3\sqrt{3})$

c) <u>Etude de la courbure</u>

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$f'(x) = \frac{x^3 - 3x^2}{\sqrt{x^3(x-2)^3}}$$

$$\left(\sqrt{x^3(x-2)^3}\right)' = \frac{3x^2(x-2)^3 + x^3 \cdot 3(x-2)^2}{2\sqrt{x^3(x-2)^3}}$$

$$= \frac{3x^2(x-2)^2\left[(x-2)+x\right]}{2\sqrt{x^3(x-2)^3}}$$

$$= \frac{3x^2(x-2)^2\,(\overbrace{2x-2}^{\cancel{2}(x-1)})}{\cancel{2}\sqrt{x^3(x-2)^3}}$$

$$= \frac{3x^{\cancel{2}}(x-2)^{\cancel{2}}(x-1)}{\cancel{x}\,\cancel{(x-2)}\,\sqrt{x(x-2)}} = \frac{3x(x-1)(x-2)}{\sqrt{x(x-2)}}$$

$$f''(x) = \frac{(3x^2-6x)\sqrt{x^3(x-2)^3} - (x^3-3x^2)\,\dfrac{3x(x-1)(x-2)}{\sqrt{x(x-2)}}}{x^3(x-2)^3}$$

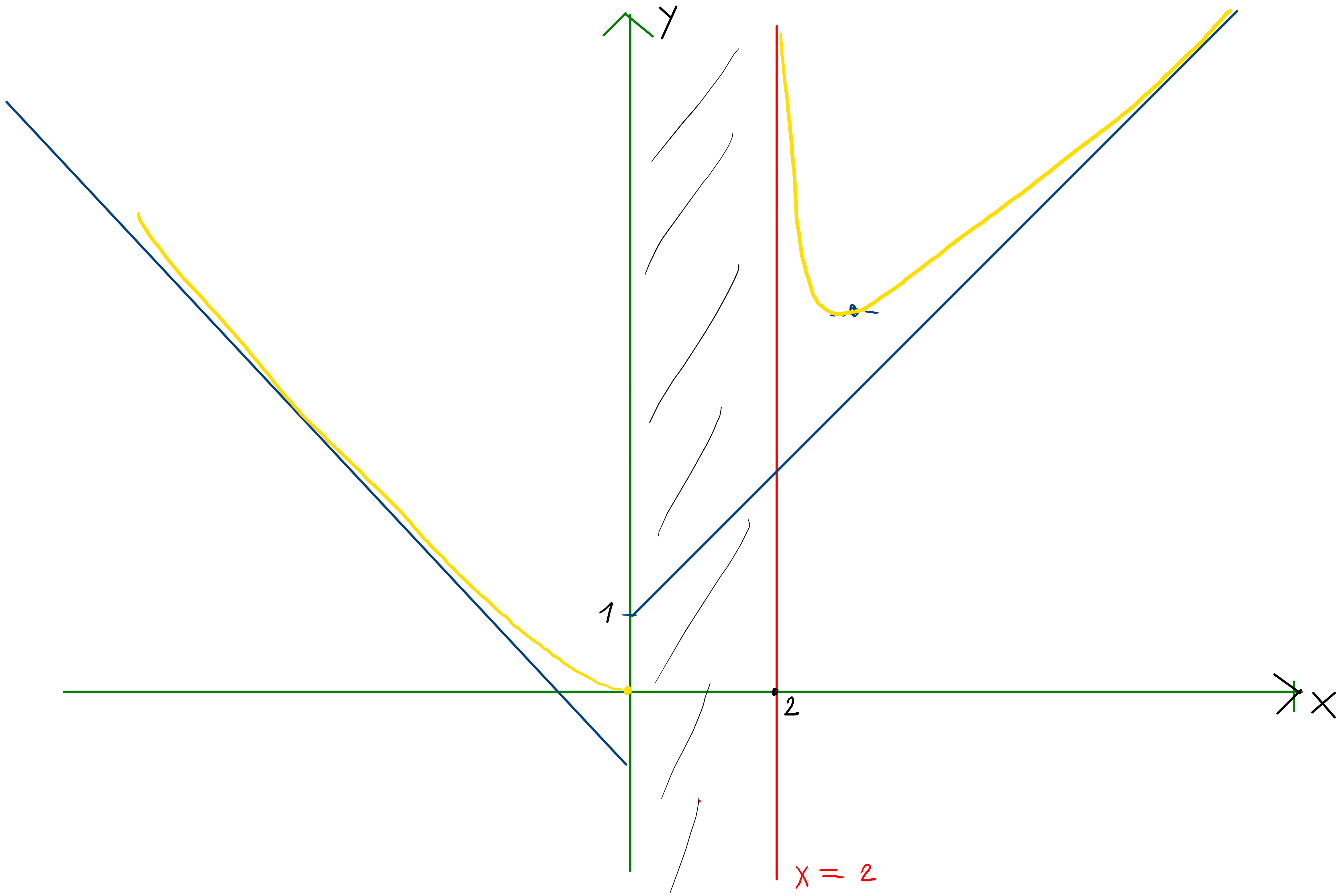
$$= \frac{\overbrace{(3x^2-6x)}^{3x(x-2)}\,x^2(x-2)^2 - \overbrace{(x^3-3x^2)}^{x^2(x-3)}\,3x(x-1)(x-2)}{x^3(x-2)^3\sqrt{x(x-2)}}$$

$$= \frac{3x^3(x-2)^3 - 3x^3(x-3)(x-1)(x-2)}{x^3(x-2)^3\sqrt{x(x-2)}} =$$

$$= \frac{3\cancel{x^3}\,\cancel{(x-2)}\left[(x-2)^2 - (x-3)(x-1)\right]}{\cancel{x^3}(x-2)^{\cancel{3}}_{2}\sqrt{x(x-2)}} = \frac{3\left[1\right]}{(x-2)^2\sqrt{x(x-2)}}$$

$\forall x \in ED(f'')$, $f''(x) > 0$ ouf!

f est convexe partout !


y
1
2
x
x = 2