$f(x) = \frac{1}{2}(x+1)^2 |x-2|$

$D_f = \mathbb{R}$

Zéros : $f(x) = 0$ $\quad \underset{\geq 0}{\frac{1}{2}} \underbrace{(x+1)^2}_{\geq 0} \underbrace{|x-2|}_{\geq 0} = 0$ $\quad x = -1$, $x = 2$

$(|a| = 0 \Leftrightarrow a = 0)$

Signe :

x		-1		2	
	$+$	0	$+$	0	$+$

Asymptotes : $\lim\limits_{x\to\infty} f(x) = +\infty$ $\quad f(x) = \frac{1}{2} \cdot \underset{\downarrow +\infty}{(x+1)^2} \underset{\downarrow +\infty}{|x-2|}$

$\Rightarrow$ ~~A.H. A.O.~~

Comme $D_f = \mathbb{R}$, ~~A.V.~~

Dérivée & croissance :

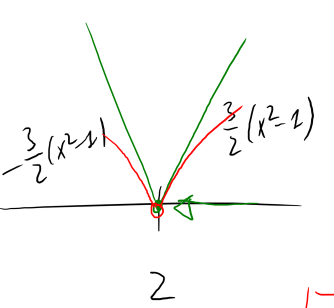
$f(x) = \frac{1}{2}(x+1)^2 |x-2| = \begin{cases} \frac{1}{2}(x+1)^2(x-2) & \text{si } x-2 \geq 0 \quad x \geq 2 \\ -\frac{1}{2}(x+1)^2(x-2) & \text{si } x-2 < 0 \quad x < 2 \end{cases}$

$\boxed{x \geq 2}$

$f'(x) = \frac{1}{2} 2(x+1)(x-2) + \frac{1}{2}(x+1)^2 \cdot 1$

$= \cancel{x^2} - x - 2 + \frac{1}{2}(\cancel{x^2} + 2x + 1)$

$= \frac{3}{2}x^2 - \frac{3}{2} = \frac{3}{2}(x^2-1) = \frac{3}{2}(x+1)(x-1)$



$\boxed{x < 2}$ $\quad f'(x) = -\left(\frac{1}{2}(x+1)^2(x-2)\right)' = -\frac{3}{2}(x+1)(x-1)$

$\lim\limits_{x \to 2, >} f'(x) = \frac{3}{2}(2+1)(2-1) = \frac{3}{2} \cdot 3 \cdot 1 = \frac{9}{2} = 4{,}5$

$\lim\limits_{x \to 2, <} f'(x) = -\frac{3}{2}(2+1)(2-1) = -\frac{3}{2} \cdot 3 \cdot 1 = -\frac{9}{2} = -4{,}5$

$\Rightarrow f'(2)$ n'existe pas

$f'(x) = 0$

$\boxed{x \geq 2}$ $\quad \frac{3}{2}(x+1)(x-1) = 0 \Leftrightarrow x = -1 \,/\, x = 1$

$\Rightarrow$ Pas de zéros de f' à droite de 2

$\boxed{x < 2}$ $\quad -\frac{3}{2}(x+1)(x-1) = 0 \Leftrightarrow x = \pm 1$ ✓

x		-1		1		2	
f'	$-$	0	$+$	0	$-$	$\|\|$	$+$
f	$\searrow$	min $(-1,0)$	$\nearrow$	max $(1,2)$	$\searrow$		$\nearrow$

f' est discontinue

$f'(x) = \begin{cases} \frac{3}{2}(x^2-1) & \text{si } x > 2 \\ -\frac{3}{2}(x^2-1) & \text{si } x < 2 \end{cases}$

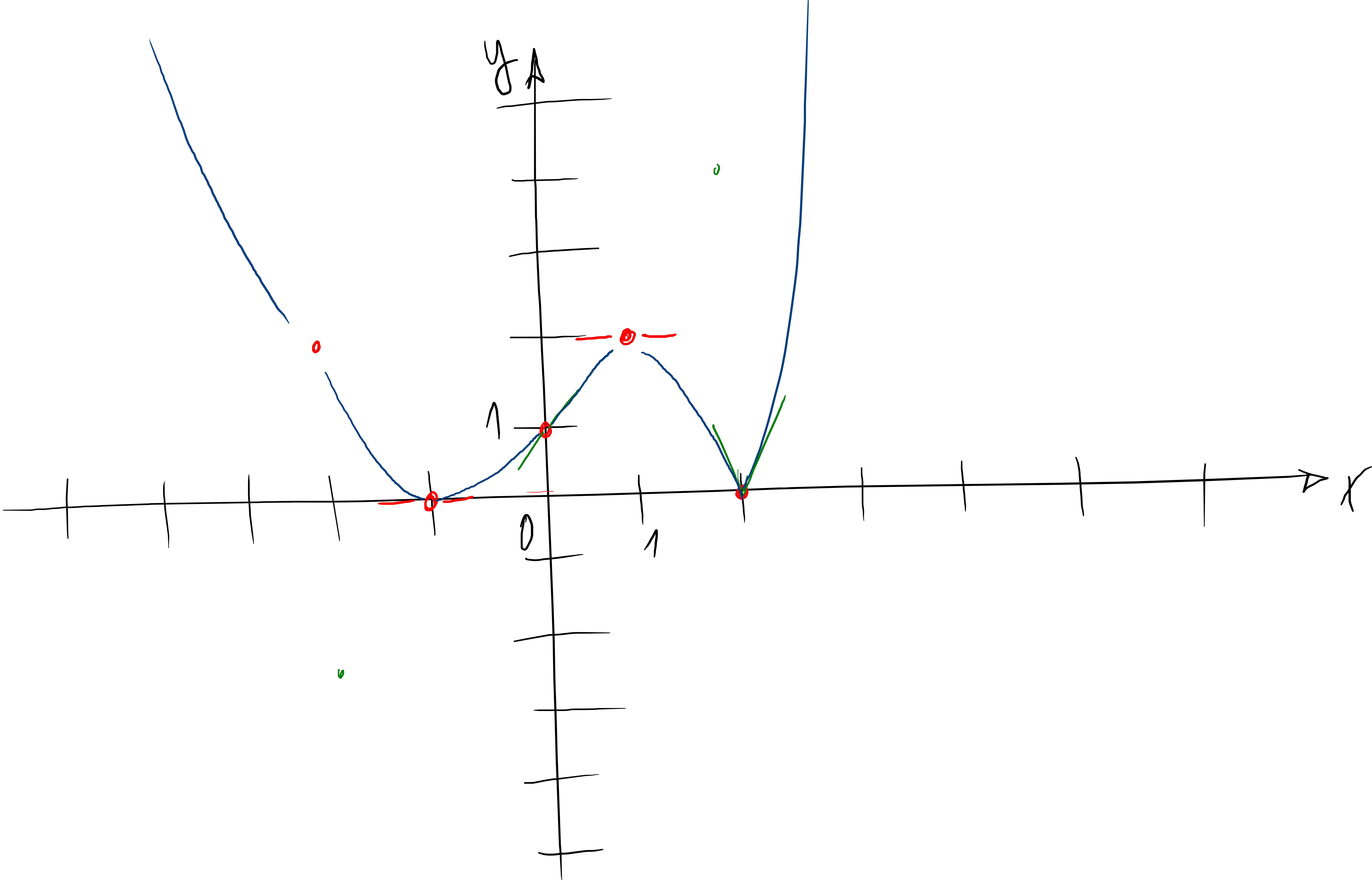
Dérivée seconde et courbure

$f''(x) = \begin{cases} 3x & \text{si } x > 2 \\ -3x & \text{si } x < 2 \end{cases}$

$f''(x) = 0 \Leftrightarrow x = 0$

x		0		2	
f''	$+$	0	$-$	$\|\|$	$+$
f	$\cup$	P.I. $(0,1)$	$\cap$		$\cup$

$f'(0) = 3/2$



y
x
1
0
1