a) $w = a + bi$

$w^2 = (a+bi)^2 = (a^2 - b^2) + 2abi$

$w^2 = 1 + 0 \cdot i \Leftrightarrow (a^2 - b^2) + 2abi = 1 + 0i$

$\Leftrightarrow \begin{cases} a^2 - b^2 = 1 \\ ab = 0 \end{cases}$

1er cas : $a = 0 \Rightarrow -b^2 = 1$

Ce qui est impossible car $b \in \mathbb{R}$

2ème cas : $b = 0 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

$\Rightarrow \quad w = \pm 1 + 0 \cdot i = \pm 1 \in \mathbb{R}$

b) $w^2 = i = 0 + i$

$\Leftrightarrow \quad a^2 - b^2 = 0 \qquad a^2 + b^2 = 1$

$2ab = 1$

$\Rightarrow \quad 2a^2 = 1 \Leftrightarrow a^2 = 1/2 \Leftrightarrow a = \pm 1/\sqrt{2}$

$\Leftrightarrow \quad a = \pm \dfrac{\sqrt{2}}{2}$. Or $b = \dfrac{1}{2a}$

$\Rightarrow \quad b = \pm \dfrac{1}{\sqrt{2}} = \pm \dfrac{\sqrt{2}}{2}$

Ce qui fait que $\omega = \pm\left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} i\right)$

c) $\omega^2 = -i = 0 - i$

$a^2 - b^2 = 0 \qquad a^2 + b^2 = 1$

$2ab = -1$

$\Rightarrow \quad a = \pm \dfrac{\sqrt{2}}{2}$ comme à la question b).

$\Rightarrow b = \mp \dfrac{\sqrt{2}}{2}$, seul le signe change. Ainsi,

$$\omega = \pm\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right)$$

d) $w^2 = -9 + 0 \cdot i$

$w = \pm 3i$ car $3^2 \cdot i^2 = -9$

e) $w^2 = 3 + 4i = (a + bi)^2$

$\Leftrightarrow \begin{cases} a^2 - b^2 = 3 \\ a^2 + b^2 = 5 \\ 2ab = 4 \end{cases}$ $\Rightarrow 2a^2 = 8$

$a = \pm 2$

$\Rightarrow \quad b = \frac{4}{2a} = \frac{2}{a} = \frac{2}{\pm 2}$

$= \pm 1$

$\Rightarrow \quad w = \pm (2 + i)$

f) $w^2 = -5 + 12i$ $\sqrt{(-5)^2 + 12^2}$

$a^2 - b^2 = -5 \,/\, a^2 + b^2 = 13 \,/\, 2ab = 12$

$\Rightarrow 2a^2 = 8 \Leftrightarrow a^2 = 4 \Leftrightarrow a = \pm 2.$

Or $b = \dfrac{6}{a} = \pm 3$

$\Rightarrow w = \pm(2 + 3i)$