$$\boxed{\text{Produit scalaire}} \qquad \vec{a}, \vec{b} \in \mathbb{R}^3$$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

déf: $$\vec{a} \cdot \vec{b} = \underbrace{a_1 b_1 + a_2 b_2 + a_3 b_3}_{\text{un nombre}} \in \mathbb{R}$$

$$\mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$$

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

déf: $$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \quad \in \mathbb{R}^n$$

déf: $$x \cdot y = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + \ldots + x_n y_n$$

Norme

longueur d'un vecteur

$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}$$

En effet,

$$\vec{a} \cdot \vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = a_1 \cdot a_1 + a_2 \cdot a_2 + a_3 \cdot a_3$$

$$= a_1^2 + a_2^2 + a_3^2$$

$$\Rightarrow \sqrt{\vec{a} \cdot \vec{a}} = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$\boxed{\text{déf: } \vec{a} \text{ est } \underline{\text{unitaire}} \text{ si } \|\vec{a}\| = 1}$$

Example:

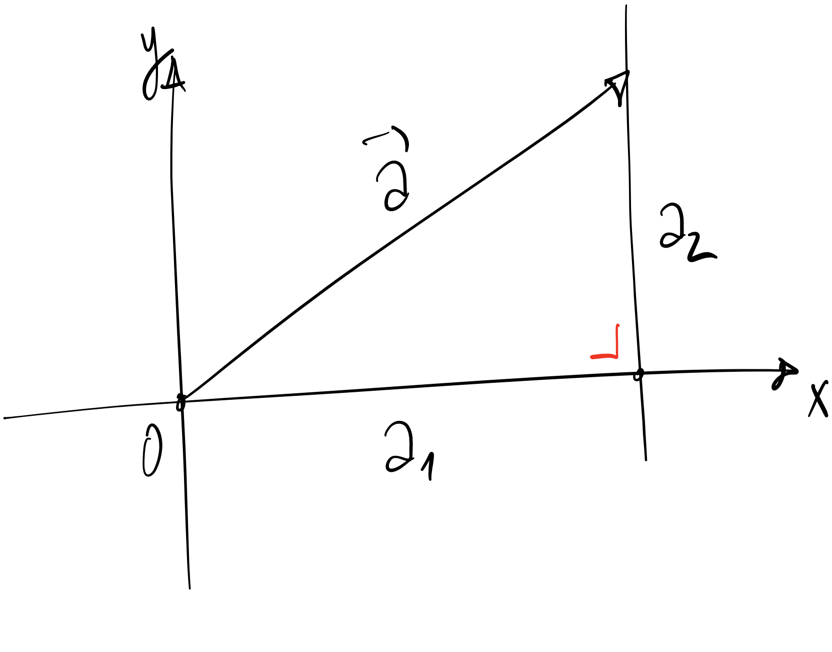
$$\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix} = -3+8 = 5$$

$$\begin{pmatrix}4\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\4\end{pmatrix} = 4-4 = 0$$

$$\begin{pmatrix}-1\\-2\end{pmatrix}\cdot\begin{pmatrix}10\\12\end{pmatrix} = -34$$

$$\left\|\begin{pmatrix}1\\4\end{pmatrix}\right\| = \sqrt{1+16} = \sqrt{17} \simeq 4{,}1$$

$$\left\|\begin{pmatrix}3\\4\end{pmatrix}\right\| = 5$$

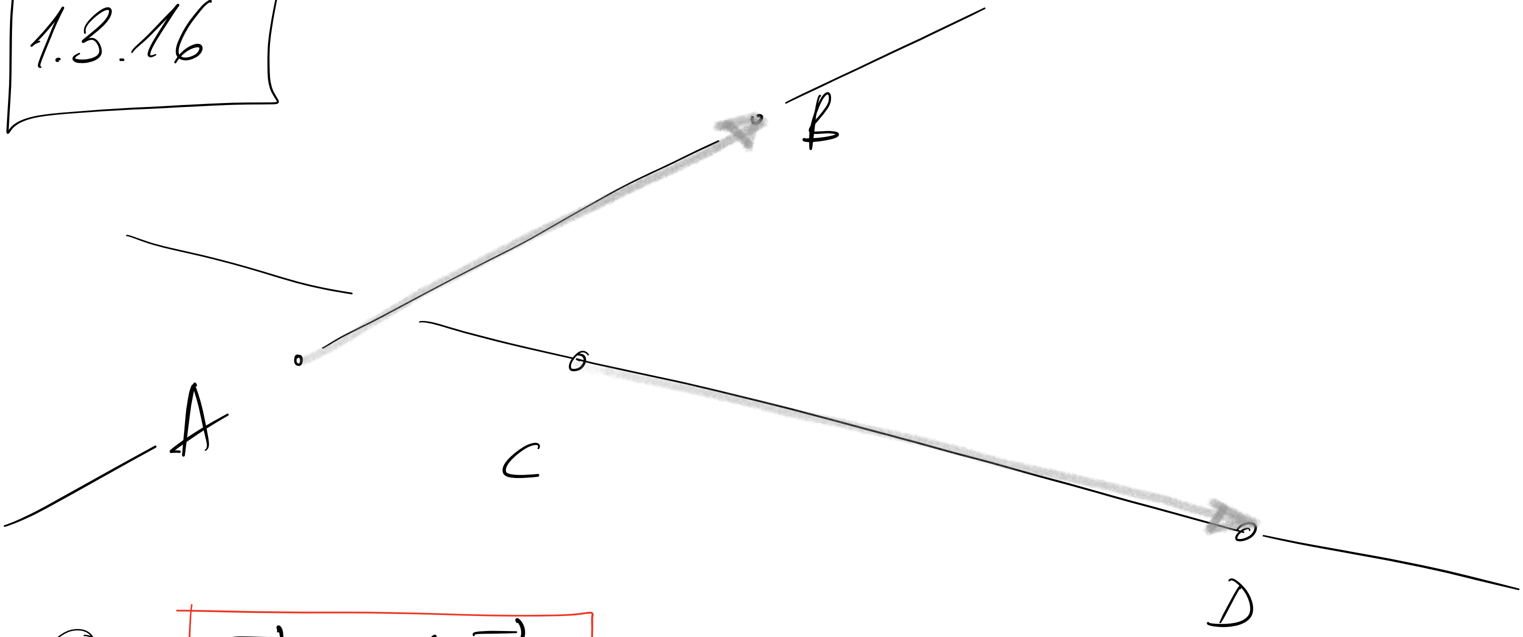


$$\|\vec a\| = \sqrt{a_1^2 + a_2^2}$$

$$= \sqrt{\vec a \cdot \vec a}$$

$$= \sqrt{a_1 \cdot a_1 + a_2 \cdot a_2}$$

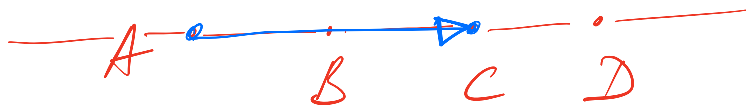
1.3.16



1. $\boxed{\vec{AB} = k\,\vec{CD}}$

$\Rightarrow$ d_{AB} et d_{CD} sont confondues si $C = A + k\vec{AB}$

ou



$d_{AB} \parallel d_{CD}$

sinon

2. $\boxed{\vec{AB} \neq k\,\vec{CD}}$

$\Rightarrow$ d_{AB} et d_{CD} sont sécantes

ou si $A + k\vec{AB} = C + m \cdot \vec{CD}$

d_{AB} et d_{CD} sont gauches

sinon