$$\boxed{\cos(x+y) = \cos x \cos y - \sin x \sin y}$$ ← donné

$$\cos(2x) = \cos x \cos x - \sin x \sin x$$

$$= \boxed{\cos^2 x - \sin^2 x} = \cos^2 x - (1 - \cos^2 x)$$

$$= 2\cos^2 x - 1 \qquad = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$\Leftrightarrow \quad 2\cos^2 x = \cos(2x) + 1$$

$$\Leftrightarrow \quad \cos^2 x = \frac{\cos(2x)+1}{2}$$

$$\Leftrightarrow \quad \boxed{\cos^2\left(\frac{\alpha}{2}\right) = \frac{\cos(\alpha)+1}{2}}$$ À démontrer

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$$\boxed{\sin^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{2}}$$ À démontrer

$$1 - 2\sin^2 x = \cos(2x) \Leftrightarrow 1 - \cos(2x) = 2\sin^2(x)$$

4.3.8 d)

$$1+\sin x = \cos 2x = \cos^2 x - \sin^2 x$$

$$1+\sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x = 0 \quad \Rightarrow \quad \begin{matrix} \sin x = 0 \\ \sin x = -\frac{1}{2} \end{matrix}$$

$$x = k\cdot\pi \Leftrightarrow \left(\begin{matrix} x = 0 + k2\pi \\ x = \pi + k2\pi \end{matrix} \right)$$

$$x = \arcsin\left(-\frac{1}{2}\right) + k2\pi \quad \Leftrightarrow \quad x = -\frac{\pi}{6} + k2\pi$$

$$x = \pi - \arcsin\left(-\frac{1}{2}\right) + k2\pi \quad \Leftrightarrow \quad x = \frac{7\pi}{6} + k2\pi$$