Vecteurs

① $a + (b + c) = (a + b) + c$

② $a + b = b + a$

③ 0 tq. $a + 0 = a$

④ Pour tout a, il existe $b = -a$
tq. $a + (-a) = 0$

⑤ $k \cdot (l \cdot a) = (k \cdot l) \cdot a$

⑥ $1 \cdot a = a$

⑦ $(k + l) \cdot a = k \cdot a + l \cdot a$

⑧ $k(a + b) = k \cdot a + k \cdot b$

Combinaison linéaire

Exemple :

vecteur

Soit $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$

scalaire

$k = 5 \quad l = 6 \quad m = -2$

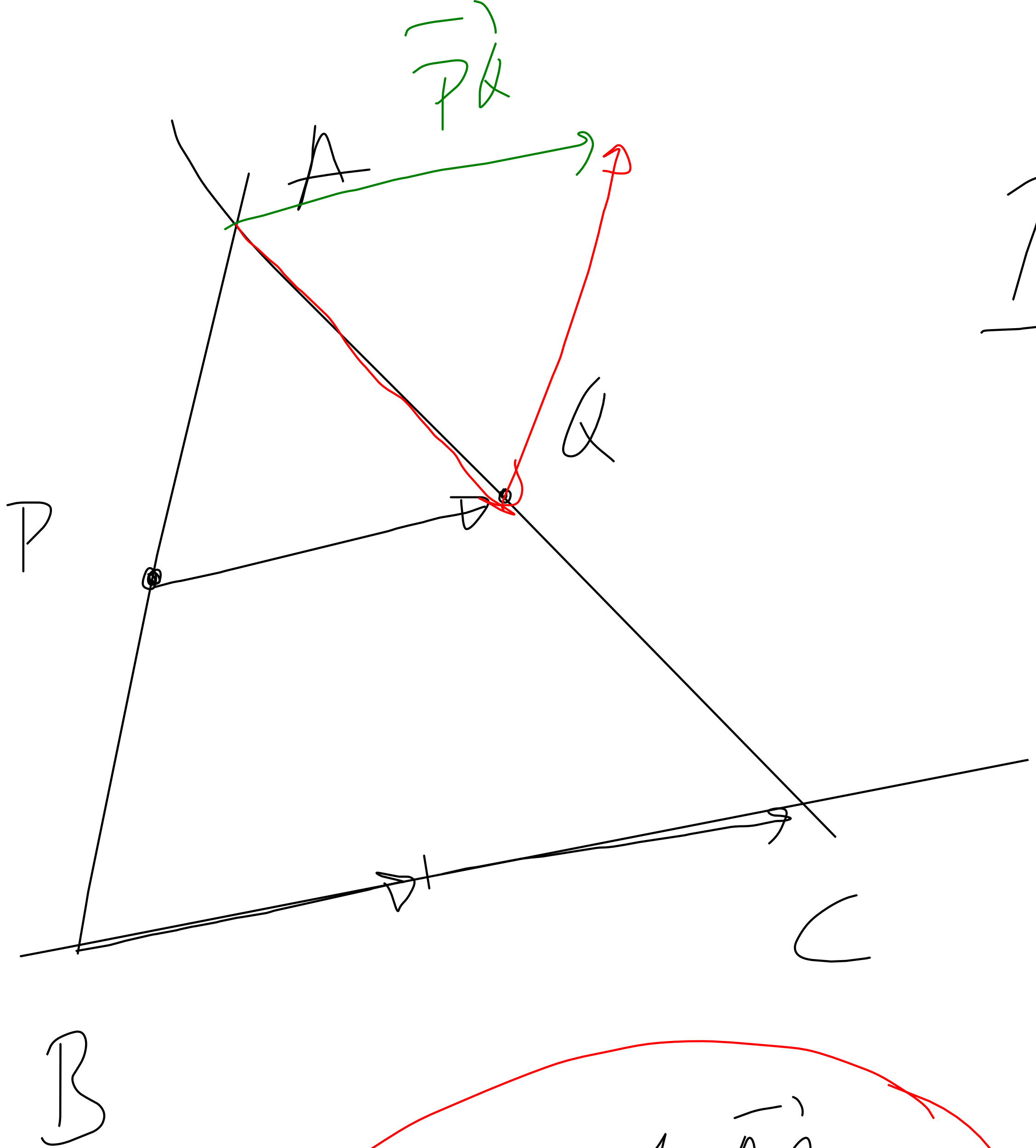
$$k \cdot \vec{a} + l \cdot \vec{b} + m \cdot \vec{c} =$$

$$5 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 6 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + (-2) \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} =$$

$$\begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \\ -6 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix} = \begin{pmatrix} 15 \\ 8 \\ 3 \end{pmatrix}$$

combinaison

linéaire



Prop. P, Q sont les milieux de AB, resp. AC.

$$\vec{PQ} = \frac{1}{2}\vec{BC}$$

preuve:

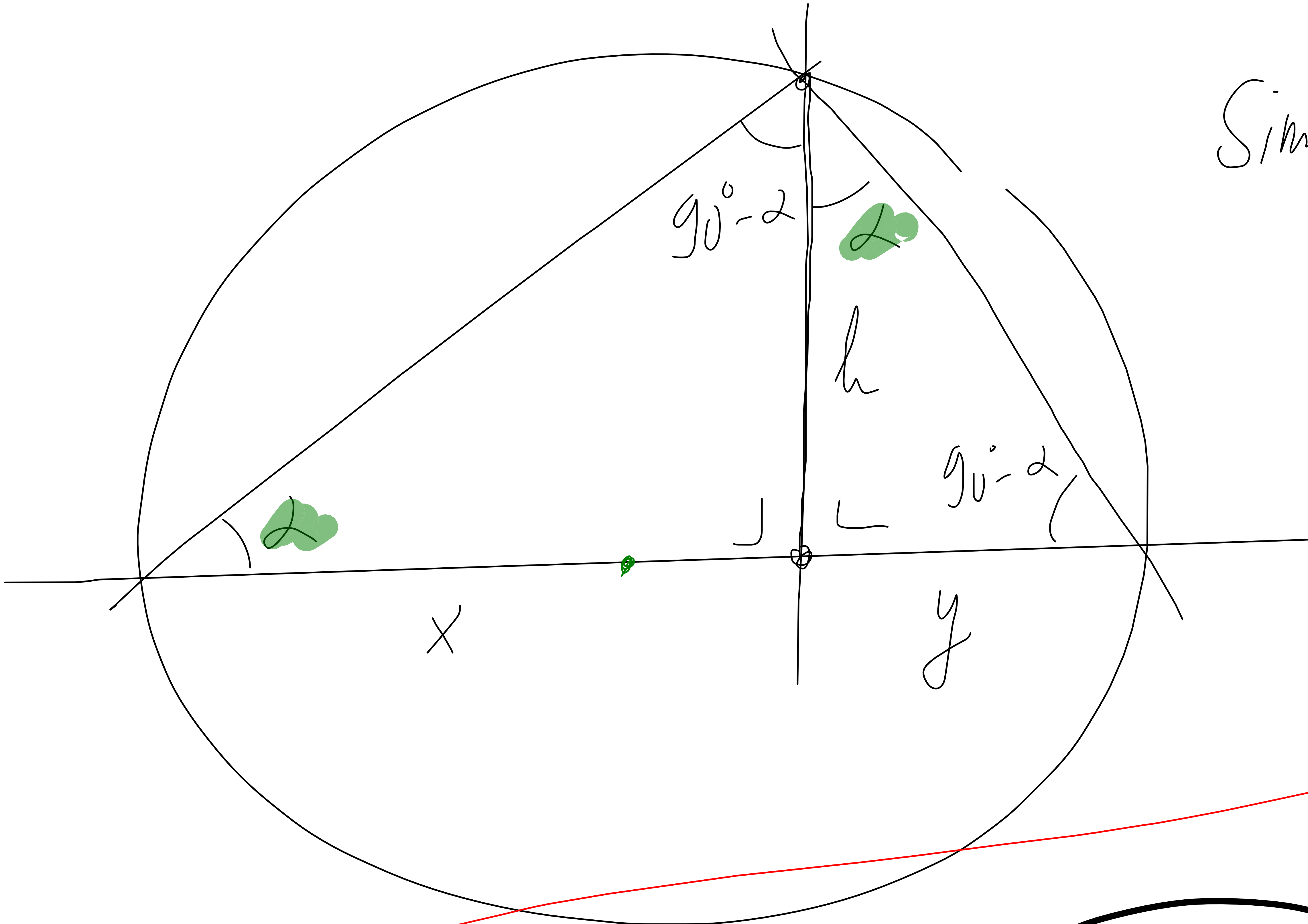
$$\vec{AP} = \frac{1}{2}\vec{AB}$$
$$\vec{AQ} = \frac{1}{2}\vec{AC}$$

Définition des milieux

$$\vec{AQ} + \vec{PA} = \boxed{\vec{AQ} - \vec{AP}} = \frac{1}{2}\vec{AC} - \frac{1}{2}\vec{AB}$$
$$= \frac{1}{2}\left(\vec{AC} - \vec{AB}\right)$$
$$= \frac{1}{2}\left(\vec{AC} + \vec{BA}\right)$$
$$= \frac{1}{2}\left(\vec{BA} + \vec{AC}\right) = \frac{1}{2}\vec{BC}$$

$$\vec{AQ} + \vec{PA} = \vec{PA} + \vec{AQ} = \vec{PQ}$$

CQFD



Similitude :

$$\frac{x}{h} = \frac{h}{y}$$

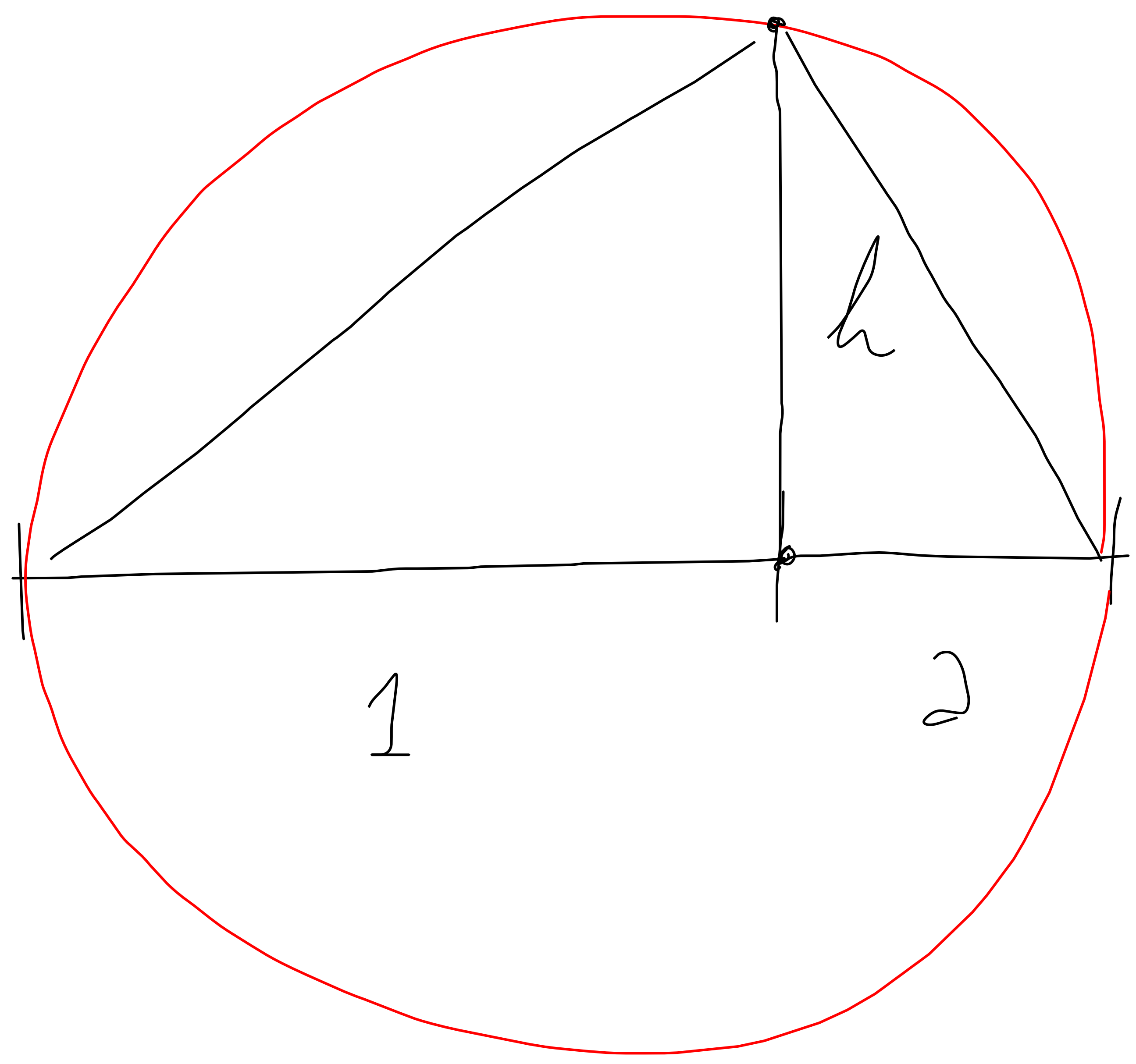
$$x = \frac{h^2}{y}$$

$$xy = h^2$$

$$xy = h^2$$

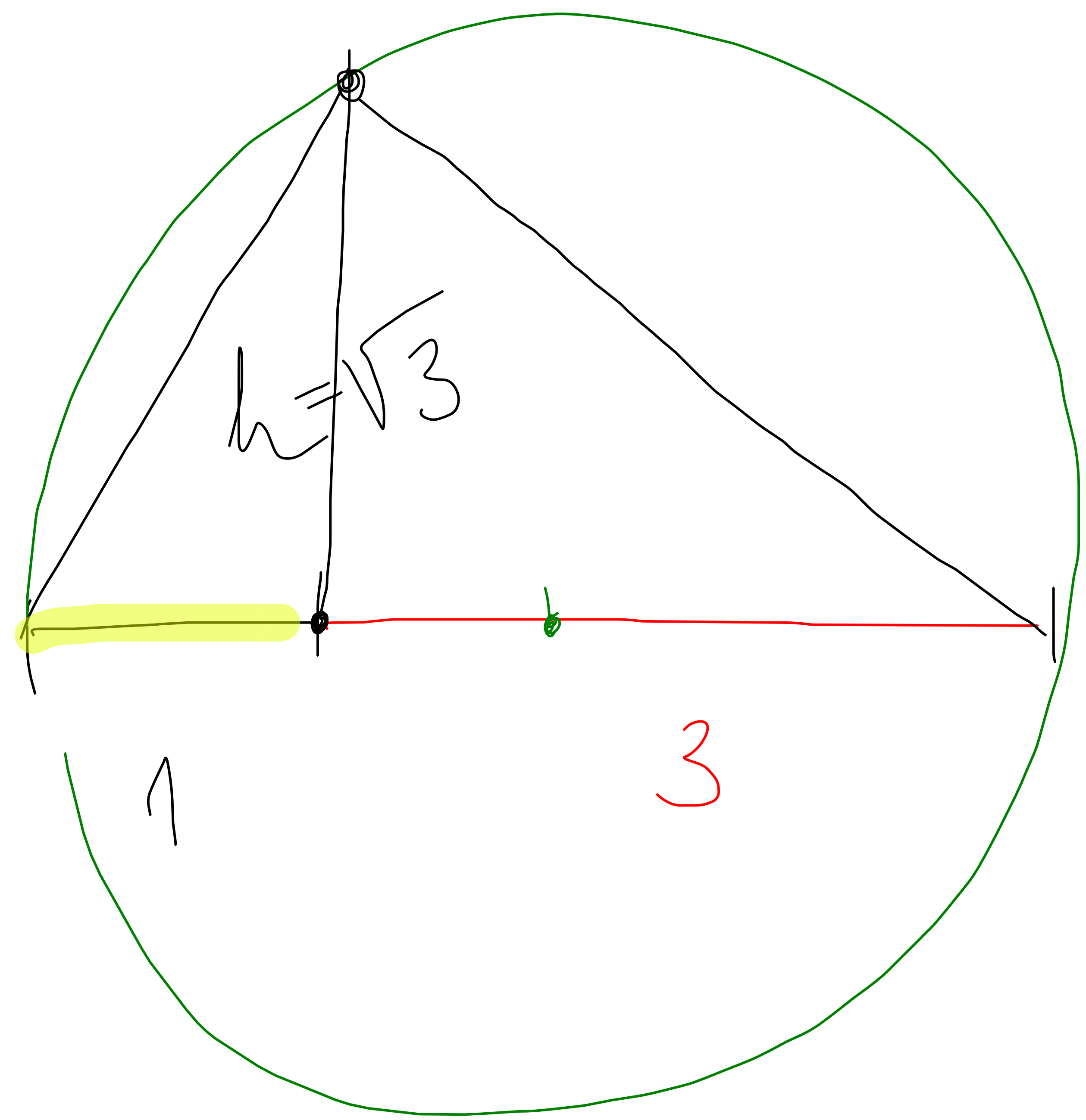
Théorème de la hauteur

Dessiner $\sqrt{2}$ à la règle et au compas



$$\Longleftarrow \quad h^2 = 2 \cdot 1$$

$$h = \sqrt{2}$$



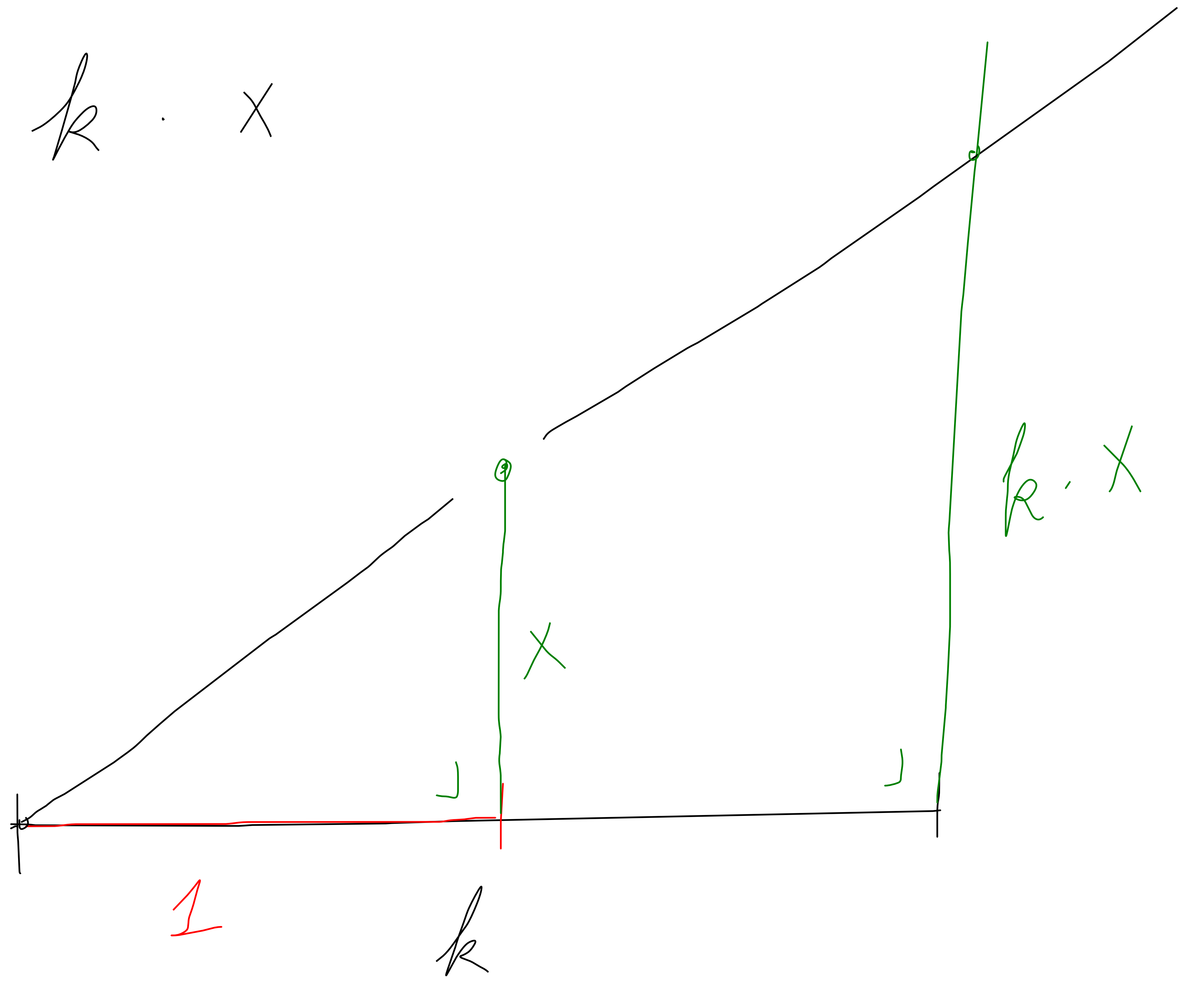
thm. de la hauteur

$$1 \cdot 3 = h^2$$

$$3 = h^2$$

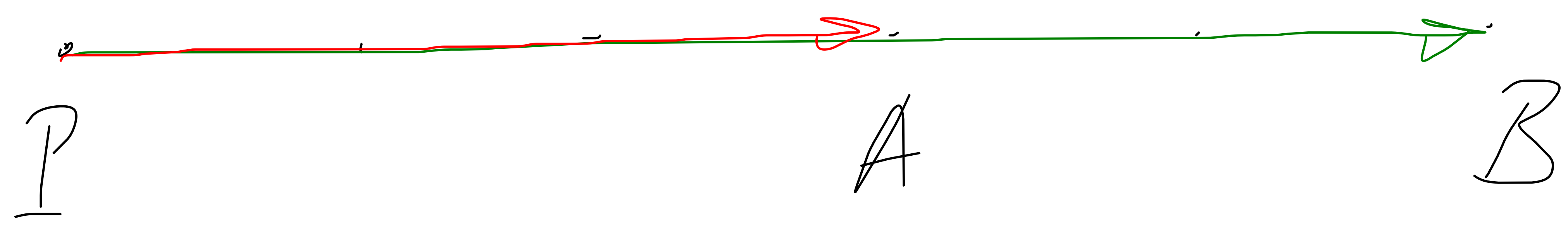
$$\sqrt{3} = h$$

$k \cdot x$



$$\overrightarrow{PA} = -\frac{3}{5}\overrightarrow{BP}$$

$$= \frac{3}{5}\overrightarrow{PB}$$



$$\vec{PA} = \frac{3}{7}\vec{AB}$$