Calculer la dérivée et simplifier le résultat le plus possible.

1) $f(x) = (x-1)(x+2)$

2) $f(x) = (x+1)^3$

3) $f(x) = (3x+2)(1-2x)$

4) $f(x) = x^2(x+1)$

5) $f(x) = (x+2)(x-2)(x+1)$

6) $f(x) = \frac{1}{x}$

7) $f(x) = \frac{1}{x^2}$

8) $f(x) = \dfrac{x+1}{x-1}$

9) $f(x) = \dfrac{1-x}{2x+2}$

10) $f(x) = \dfrac{2x+6}{cx+d}$ par rapport à x

a, b, c, d fixés

11) $f(x) = \dfrac{1}{x^2+1}$

12) $f(x) = \dfrac{x}{x^2+x+1}$

13) $f(x) = \dfrac{x^2+2x+1}{x^2+3x-2}$

14) $f(x) = \dfrac{3x^2-4x+5}{2x^2+x-3}$

1) $f(x) = x^2 + 2x - x - 2$

$= x^2 + x - 2$

$f'(x) = (x^2)' + (x)' - 2 \cdot (1)' = 2x + 1 - 0 = 2x + 1$

2) $f(x) = x^3 + 3x^2 + 3x + 1$

$f'(x) = (x^3)' + 3(x^2)' + 3(x)' + (1)'$

$= 3x^2 + 3 \cdot 2x + 3 \cdot 1 = 3x^2 + 6x + 3$

3) $f(x) = (3x + 2)(1 - 2x) = 3x - 6x^2 + 2 - 4x$

$= -6x^2 - x + 2$

$f'(x) = -6(x^2)' - (x)' = -6 \cdot 2x - 1 = -12x - 1$

4) $f(x) = x^2(x+1) = x^3+x^2$

$f'(x) = 3x^2+2x$

5) $f(x) = (x^2-4)(x+1) = x^3+x^2-4x-4$

$f'(x) = 3x^2+2x-4$

6) $f'(x) = \frac{(1)'\cdot x - 1\cdot(x)'}{x^2} = \frac{0\cdot x - 1}{x^2} = -\frac{1}{x^2}$

7) $f'(x) = (x^{-2})' = -2x^{-3} = \frac{-2}{x^3}$

$= \left(\frac{1}{x^2}\right)' = \frac{1'\cdot x^2 - 1\cdot(x^2)'}{(x^2)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$

8) $f'(x) = \frac{(x+1)'(x-1) - (x+1)(x-1)'}{(x-1)^2} = \frac{1\cdot(x-1) - (x+1)\cdot 1}{(x-1)^2}$

$= \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}$

9) $$f'(x) = \left(\frac{1-x}{2x+2}\right)' = \frac{(1-x)'(2x+2) - (1-x)(2x+2)'}{(2x+2)^2}$$

$$= \frac{(-1)\cdot(2x+2) - (1-x)\cdot 2}{(2x+2)^2}$$

$$= \frac{-2x-2-2+2x}{(2x+2)^2} = \frac{-4}{(2x+2)^2}$$

10) $$f'(x) = \frac{(ax+b)'(cx+d) - (ax+b)(cx+d)'}{(cx+d)^2}$$

$$= \frac{a(cx+d) - (ax+b)\cdot c}{(cx+d)^2} = \frac{acx+ad-acx-bc}{(cx+d)^2}$$

$$= \frac{ad-bc}{(cx+d)^2}$$

11) $$f'(x) = \frac{0\cdot(x^2+1) - 1\cdot 2x}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$$

12) $$f'(x) = \frac{1\cdot(x^2+x+1) - x\cdot(2x+1)}{(x^2+x+1)^2} = \frac{-x^2+1}{(x^2+x+1)^2}$$

13) $f'(x) = \dfrac{(2x+2)(x^2+3x-2) - (x^2+2x+1)(2x+3)}{(x^2+3x-2)^2}$

$= \dfrac{2x^3+6x^2-4x+2x^2+6x-4 - (2x^3+4x^2+2x+3x^2+6x+3)}{(x^2+3x-2)^2}$

$= \dfrac{x^2-6x-7}{(x^2+3x-2)^2}$

14) $f'(x) = \dfrac{(6x-4)(2x^2+x-3) - (3x^2-4x+5)(4x+1)}{(2x^2+x-3)^2}$

$= \dfrac{11x^2-38x+7}{(2x^2+x-3)^2}$