Signe de la fonction donnée par

$$\boxed{f(x) = \ln(1-x-x^2)}$$

D_f, zéros & signe

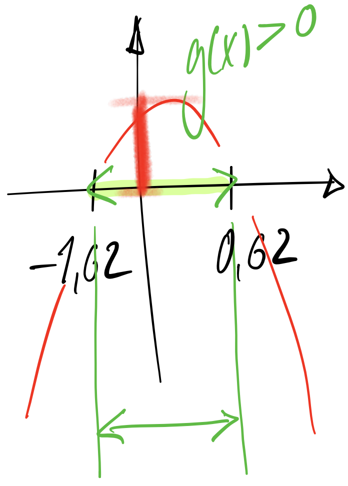
$D_{\ln} =]0;+\infty[$

D_f est donné par le signe de $g(x) = 1-x-x^2$

$$1-x-x^2=0 \Leftrightarrow x = \frac{1\pm\sqrt{1+4}}{-2} = \frac{1\pm\sqrt{5}}{-2} \begin{cases} -1{,}618 \\ +0{,}618 \end{cases}$$

$-x^2-x+1=0$

x		$-1{,}62$		$0{,}62$	
	$-$	0	$+$	0	$-$



$$\Rightarrow D_f =]-1{,}62;0{,}62[$$

Zéros : $\ln(1-x-x^2)=0$

$\ln x = u \Leftrightarrow x = e^u$

$\Leftrightarrow 1-x-x^2=1$

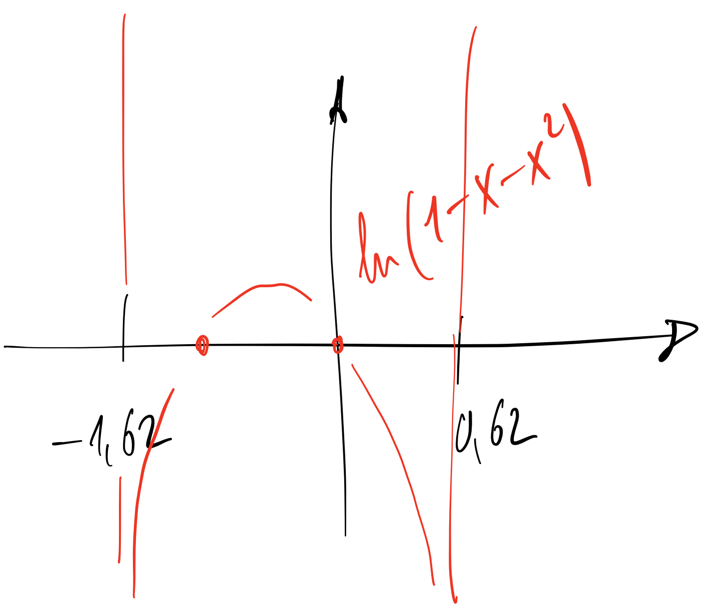
$\Leftrightarrow \quad x^2+x=0 \quad \Leftrightarrow \quad \boxed{x=0} \quad \boxed{x=-1} \quad \in D_f$ ✓

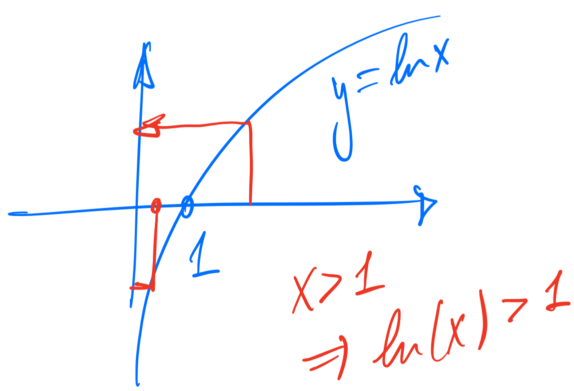
Signe:

x		$-1{,}618$		-1		0		$0{,}618$	
	//////	‖	$-$	0	$+$	0	$-$	‖	//////

$$f(-0{,}5) = \ln\left(1-(-0{,}5)-(-0{,}5)^2\right) = \ln(1+0{,}5-0{,}25)$$

$$= \ln(1{,}25) > 0$$





$$\boxed{1.3.2}$$

b) $z^4+1 = (z^2+i)(z^2-i)$

ambigu

$$\ll = (z+i\sqrt{i})(z-i\sqrt{i})(z+\sqrt{i})(z-\sqrt{i}) \gg$$

$$(a^2-b^2)+2abi$$

$$\sqrt{i} = \sqrt{0+1i} \qquad (a+bi)^2 = 0+1i$$

$$a^2+b^2 = 1 \qquad |a+bi|^2 = |0+1i|$$

$$\left.\begin{aligned} a^2-b^2 &= 0 \\ 2ab &= 1 \end{aligned}\right\} \qquad (a^2-b^2)+2abi = 0+1i$$

$$2a^2 = 1 \qquad a = \pm\frac{\sqrt{2}}{2}$$

$$2b^2 = 1 \qquad b = \pm\frac{\sqrt{2}}{2}$$

$$\pm\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$$

$$\pm i\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = \pm\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)$$

$$\Rightarrow z^4+1 = \left(z-\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\right)\left(z+\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\right)\cdot$$

$$\left(z-\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right)\left(z+\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right)$$

$$(z^4+1) = (z-z_1)(z-z_2)(z-z_3)(z-z_4)$$