$P(n)$

$P(0), P(1), \dots$ à vérifier pour $n=0$, ou $n=1$... 1

$\boxed{P(n)} \Rightarrow \boxed{P(n+1)}$ 2

Prop. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = P(n)$$

preuve 1 amorçage $\boxed{n=1}$ $$\sum_{i=1}^{1} i = 1 \overset{?}{=} \frac{1(1+1)}{2} = 1$$

2 $\boxed{n \Rightarrow (n+1)}$

hyp. de réc. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1) = \frac{n(n+1)}{2} + (n+1) \overset{?}{=} \frac{(n+1)((n+1)+1)}{2}$$

$$= \frac{n(n+1) + 2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

$$= \frac{(n+1)((n+1)+1)}{2} = P(n+1)$$

CQFD

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$\boxed{n=1}$ $\quad 1 \overset{?}{=} \frac{1 \cdot 2 \cdot 3}{6}$ ✓

$\boxed{n=2}$ $\quad 1+4 = \frac{2 \cdot 3 \cdot 5}{6} = 5$ ✓ $\quad$ 1

2

$$\sum_{k=1}^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n+1)^2$$

hyp. de réc

$$\overset{\downarrow}{=} \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)}{6}$$

$$\overset{?}{=} \frac{(n+1)\ (n+1)+1)\ (2(n+1)+1)}{6}$$

$$\frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)(n+1)}{6} =$$

$$\frac{n(n+1)(2n+1) + 6(n+1)(n+1)}{6} =$$

$$\frac{(n+1)(2n^2+n+6n+6)}{6} = \frac{(n+1)(2n^2+7n+6)}{6}$$

$$= \frac{(n+1)(2n+3)(n+2)}{6}$$

$$= \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}$$

CQFD