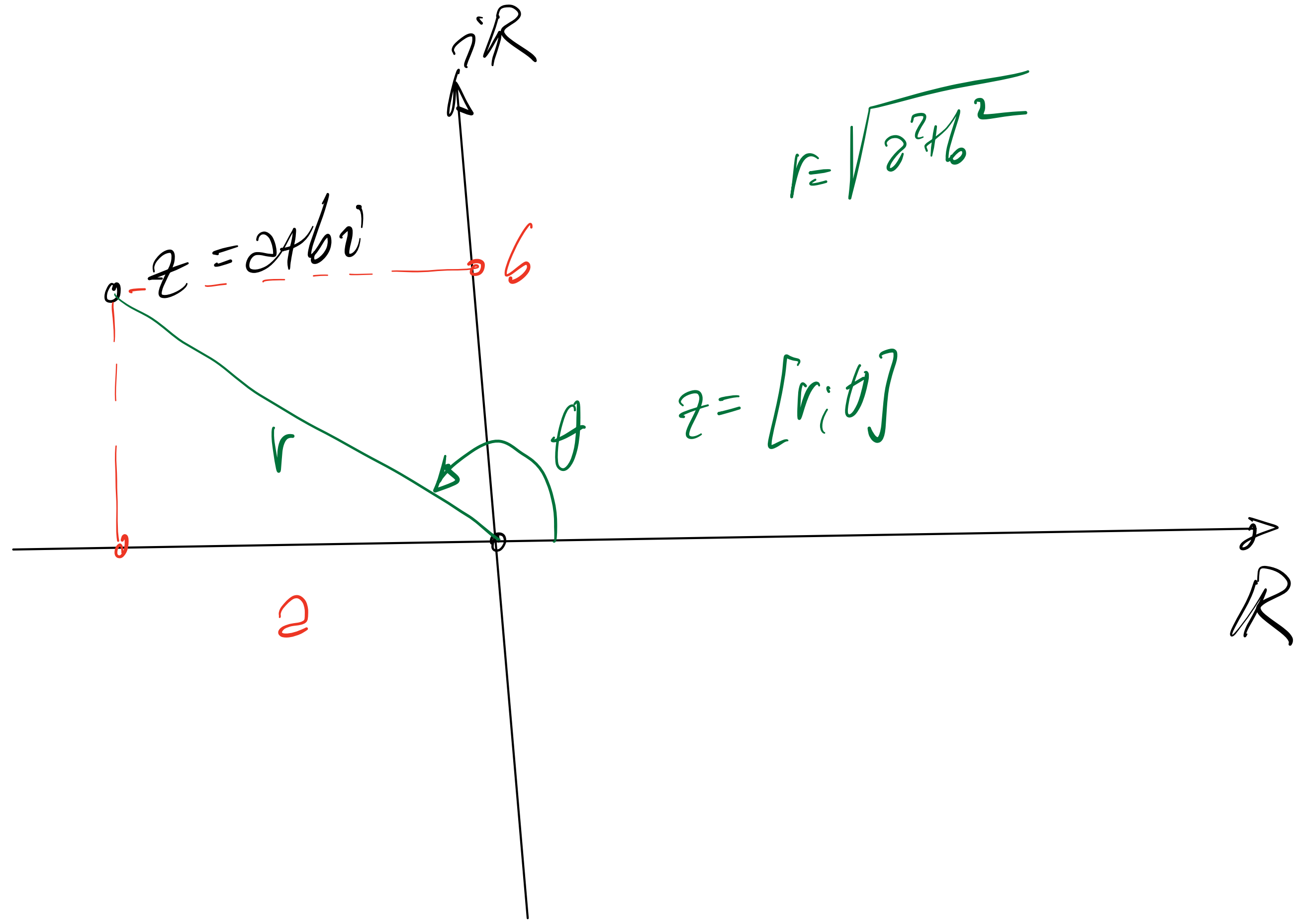
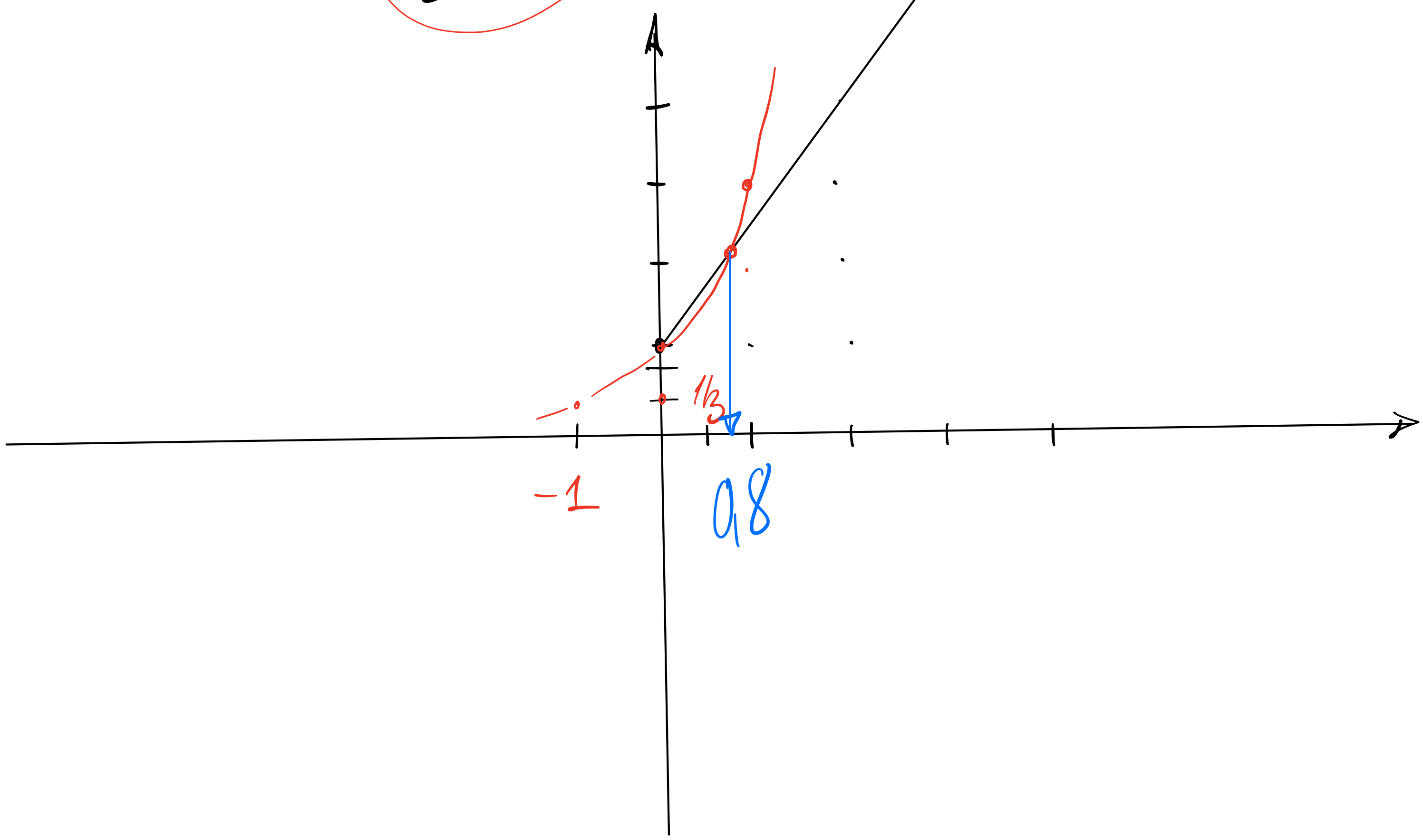


Thm : $[r;\theta]^n = [r^n;n\theta]$

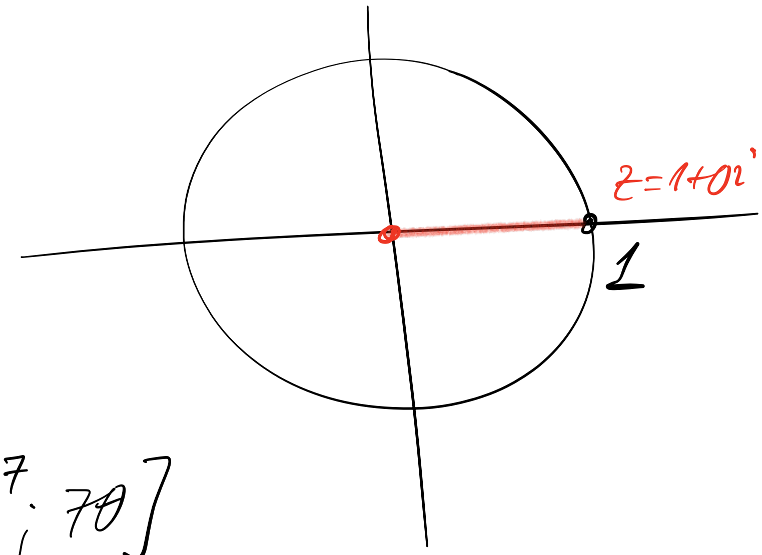
TE octobre

preuve par réc sur n

$$3^x = \frac{3}{2}x + 1$$



$1 = [1 ; 0]$



$w = [r ; \theta]$

$t.q. \quad w^7 = [1; 0] = [r^7 ; 7\theta]$

$r^7 = 1 \iff r = 1$

$7\theta = 0 + k \cdot 2\pi \qquad k = 0, 1, 2, \ldots, 6 \qquad \text{modulo } 7$

$\theta_0 = 0 \;\Big/\; \theta_1 = \frac{2\pi}{7} \;\Big/\; \theta_2 = \frac{4\pi}{7} \;\Big/\; \ldots \;\Big/\; \theta_6 = \frac{12\pi}{7}$

$$z^2 + 2\bar{z} + 5 = 0 + 0i$$

$$(x^2 - y^2) + 2xyi + 2x - 2yi + 5 = 0 + 0i$$

$$\underbrace{(x^2 - y^2 + 2x + 5)}_{Re} + \underbrace{(2xy - 2y)}_{Im}\, i = 0 + 0 \cdot i$$

$$\begin{cases} x^2 - y^2 + 2x + 5 = 0 \\ 2xy - 2y = 0 \end{cases}$$

4.2.7 b)

$$\log(x) - \log(y) = 1$$

$x>0$ et $y>0$

$$xy = 2$$

$y = \frac{2}{x}$ donne $\log(x) - \log\left(\frac{2}{x}\right) = 1$

$$\Leftrightarrow \log\left(x \cdot \frac{x}{2}\right) = 1$$

$10^{(\ )}$

$$\log(a) - \log(b) = \log\left(a \cdot \frac{1}{b}\right)$$

$$= \log\left(\frac{a}{b}\right)$$

$$\Leftrightarrow \frac{x^2}{2} = 10$$

$$\Leftrightarrow x^2 = 20 \Leftrightarrow x = 2\sqrt{5}$$

car $x>0$

$$\boxed{\log(x) = u \Leftrightarrow x = 10^u}$$

et $y = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$

$z = x + yi$

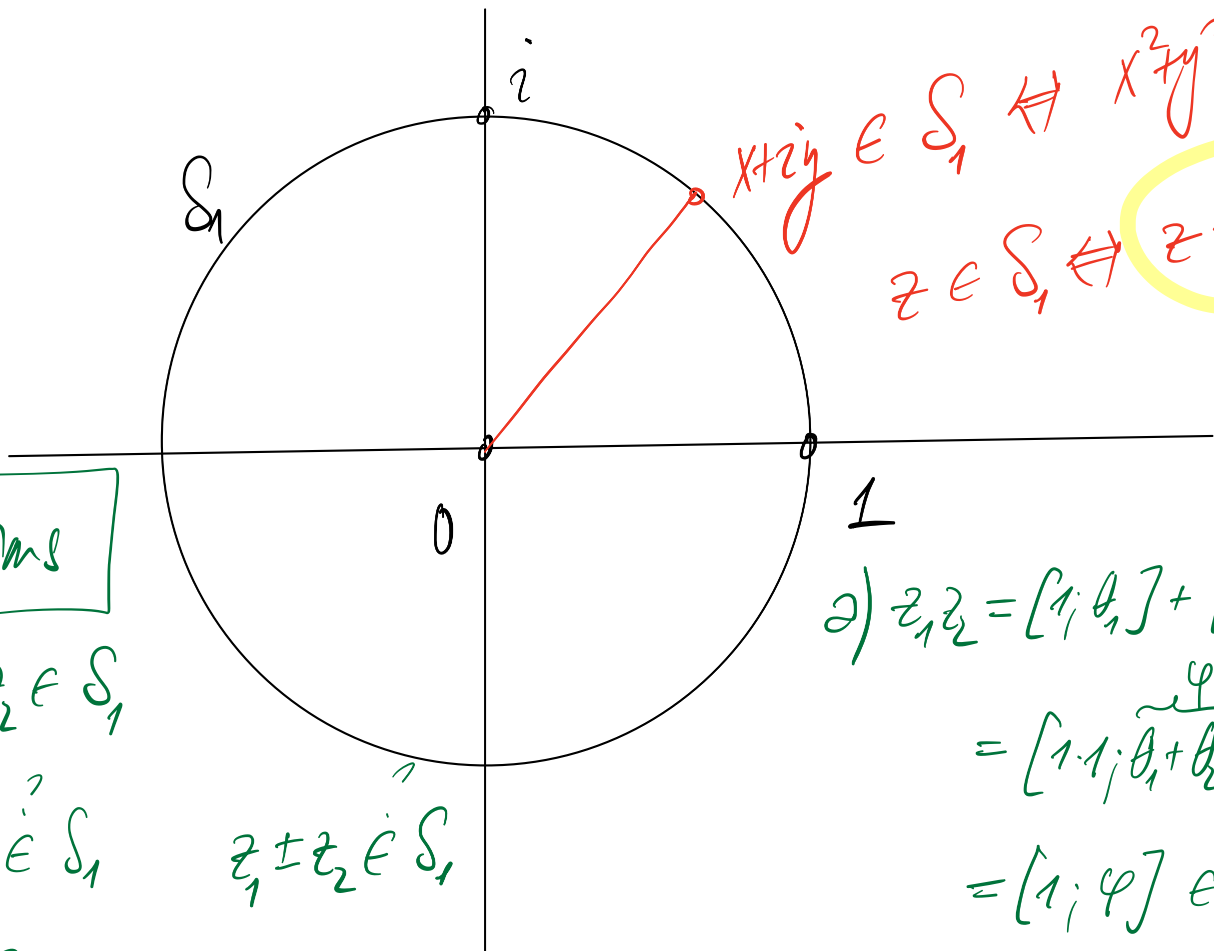
$\overline{z} + 1 = x - yi + 1 = (x+1) - yi$

$\Im(\overline{z}+1) = -y$

$\Im(a+bi) = b$ | $\Im : \mathbb{C} \to \mathbb{R}$, $a+bi \mapsto b$

$\Re(a+bi) = a$

$\Re : \mathbb{C} \to \mathbb{R}$, $a+bi \mapsto a$



$x+iy \in S_1 \Leftrightarrow x^2+y^2 = 1$

$z \in S_1 \Leftrightarrow z = [1;\theta]$, $\theta \in [0;2\pi[$

Questions

$z_1, z_2 \in S_1$

a) $z_1 z_2 \overset{?}{\in} S_1$

$z_1 \pm z_2 \overset{?}{\in} S_1$

$\frac{z_1}{z_2} \overset{?}{\in} S_1$

a) $z_1 z_2 = [1;\theta_1] + [1;\theta_2]$

$= [1\cdot 1; \overbrace{\theta_1+\theta_2}^{\varphi}]$

$= [1;\varphi] \in S_1$

$$z^4 = 1+i = [\sqrt{2}; \frac{\pi}{4}]$$

$$z = [r; \theta] \Rightarrow z^4 = [r^4; 4\theta]$$

$$\Rightarrow r^4 = \sqrt{2} \Rightarrow r = \sqrt[4]{\sqrt{2}}$$

$$4\theta = \frac{\pi}{4} + k \cdot 2\pi \qquad k = 0, 1, 2, 3$$

$$\theta_0 = \frac{\pi}{16}$$

$$\theta_1 = \frac{\pi}{16} + 1 \cdot \frac{2\pi}{4} = \frac{\pi}{16} + \frac{\pi}{2} = \frac{9\pi}{16}$$

$$\theta_2 =$$

$$\theta_3 =$$