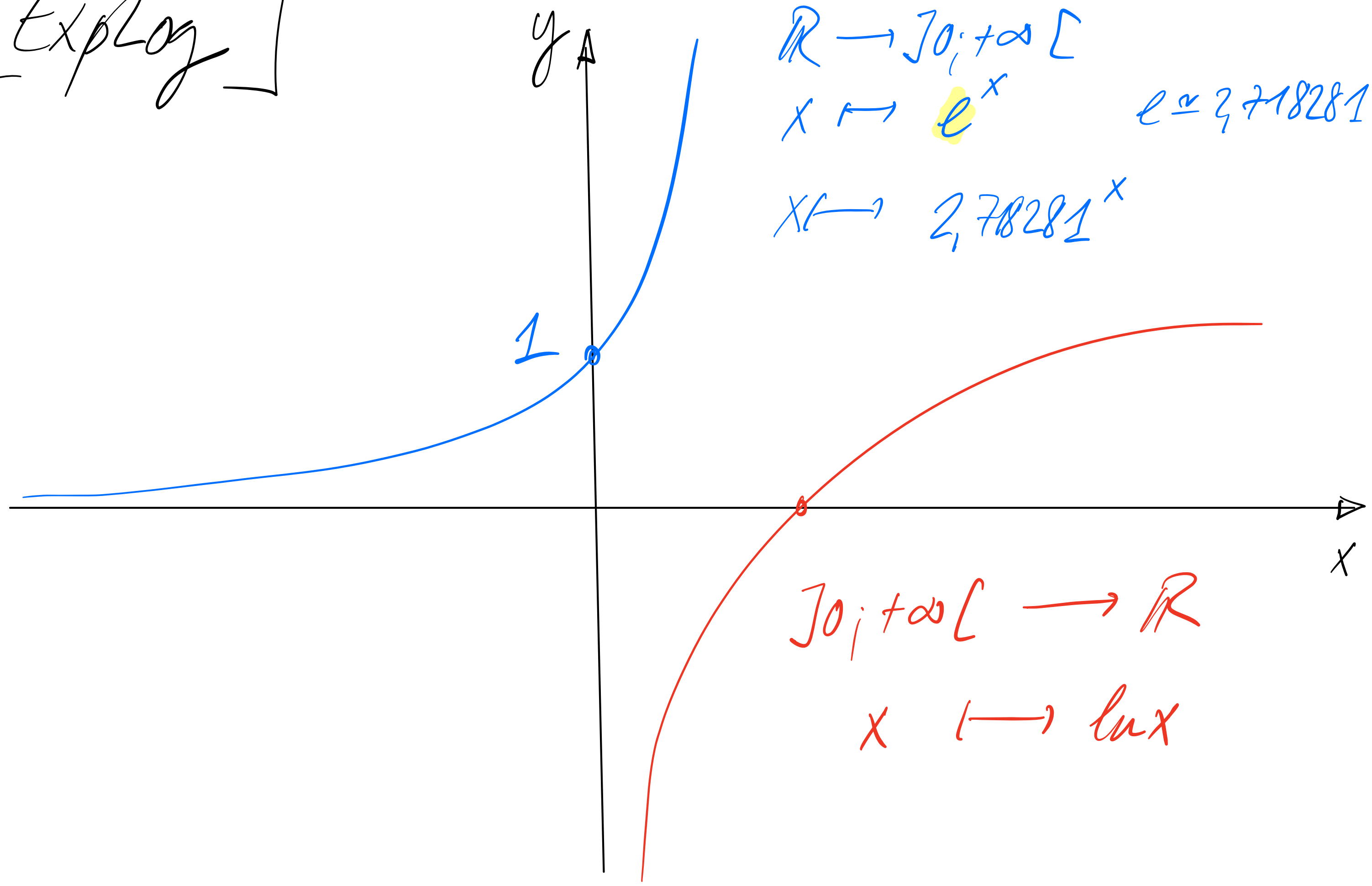
$\boxed{\text{ExpLog}}$

$\mathbb{R} \longrightarrow]0;+\infty[$

$x \longmapsto e^{x}$ $\qquad e \simeq 2{,}718281\ldots$

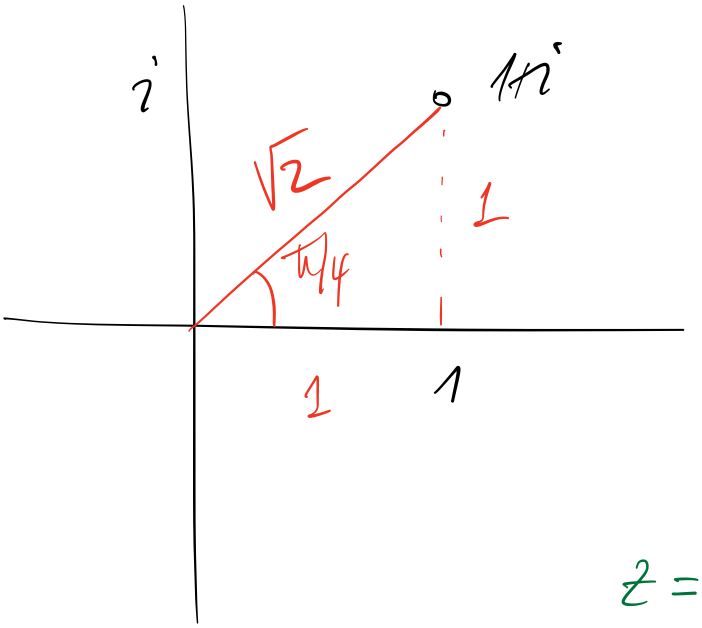
$x \longmapsto 2{,}718281^{x}$



$]0;+\infty[\longrightarrow \mathbb{R}$

$x \longmapsto \ln x$

$\boxed{1.2.8}$ $z^4 = 1+i = \left[\sqrt{2}; \frac{\pi}{4}\right] = \left[r^4; 4\theta\right]$



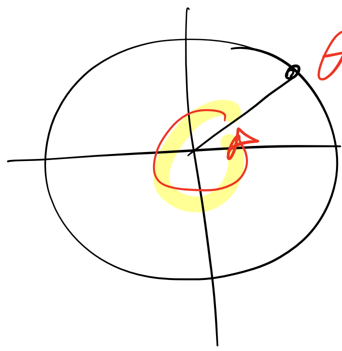
$\arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$z = [r; \theta]$ $\qquad$ $z^4 = [r^4; 4\theta]$

$\Rightarrow r^4 = \sqrt{2} \Rightarrow r = \sqrt[4]{\sqrt{2}} = \sqrt[4]{\sqrt[2]{2}} = \sqrt[8]{2}$

$4\theta = \frac{\pi}{4} + k2\pi$

$k \in \mathbb{Z}$



$\theta, \ \theta + 2\pi,$
$\theta + 4\pi,$
$\theta + 6\pi,$
$\vdots$

$\theta = \frac{\pi}{16} + k \cdot \frac{2\pi}{4} = \frac{\pi}{16} + k \cdot \frac{\pi}{2}$

k	0	1	2	3	4
θ	$\frac{\pi}{16}$	$\frac{\pi}{16} + \frac{\pi}{2}$	$\frac{\pi}{16} + \pi$	$\frac{\pi}{16} + \frac{3\pi}{2}$	$\frac{\pi}{16} + 2\pi$

$\Rightarrow k \in \mathbb{Z}_4 = \{0; 1; 2; 3\}$

$$\Rightarrow \quad z_1 = \left[\sqrt[8]{2}\ ;\ \frac{\pi}{16}\right]$$

$$z_2 = \left[\sqrt[8]{2}\ ;\ \frac{9\pi}{16}\right]$$

$$z_3 = \left[\sqrt[8]{2}\ ;\ \frac{17\pi}{16}\right]$$

$$z_4 = \left[\sqrt[8]{2}\ ;\ \frac{25\pi}{16}\right]$$

$e \simeq 2{,}71828182\ldots$

$x \mapsto e^x$

$x \mapsto \ln x = \log_e x$

TI 80

$\ln(e^2) = \log_e e^2 = 2$

$$\boxed{\log_2 x = u \iff x = 2^u}$$

$\log_{10} 0{,}1 = -1$

$\log_3 27 = 3$

$\log_{0{,}2} 5 = \log_{0{,}2} \left(\frac{1}{5}\right)^{-1}$

$= \log_{0{,}2} 0{,}2^{-1} = -1$

$\boxed{4.2.1}$

$at^2+bt+c=0$

$2^x=2^y$

$\Leftrightarrow x=y$

$$5\cdot 5^{4x-7}-120\cdot 5^{2x-3}=625$$

$$\cancel{5^1}\cdot 5^{4x-6}\cdot \cancel{5^{-1}}-120\cdot 5^{2x-3}=625$$

$$\left(5^{2x-3}\right)^2-120\, 5^{2x-3}-625=0$$

$t=5^{2x-3}$

$$t^2-120\cdot t-625=0$$

Racine carrée de $z \in \mathbb{C}$

On cherche $w \in \mathbb{C}$ tq.

$$z = a+bi = w^2$$

$$w = x+iy$$

$$w^2 = (x^2-y^2) + 2xyi$$

$$\Rightarrow \quad a = x^2 - y^2 \quad (1)$$

$$b = 2xy \quad (2) \qquad y = \frac{b}{2x}$$

$$\sqrt{a^2+b^2} = x^2+y^2 \quad (3)$$

(1)+(3) donne

$$a + \sqrt{a^2+b^2} = 2x^2$$

Truc: $z = w^2 \Rightarrow |z| = |w^2| = |w|^2$

résultat du cours

$$\Rightarrow \sqrt{a^2+b^2} = x^2+y^2$$

$$\log_2(xy) = \log_2 x + \log_2 y$$

$$\boxed{\log_2\left(\frac{x}{y}\right) = \log_2 x - \log_2 y}$$

$$\log_2(x^r) = r \log_2$$

$$\log_2 b = \frac{\log_c b}{\log_c 2}$$

$$\log(200) - \log(2) =$$

$$\log\left(\frac{200}{2}\right) = \log 100$$

$$= \log_{10} 100$$

$$= \log_{10} 10^2 = 2$$

$$S_1 = \{ a+bi \mid \sqrt{a^2+b^2} = 1 \} = \{ \cos\theta + i\sin\theta \mid \theta \in [0;2\pi[\}$$

$$\overline{a+bi} = a-bi$$

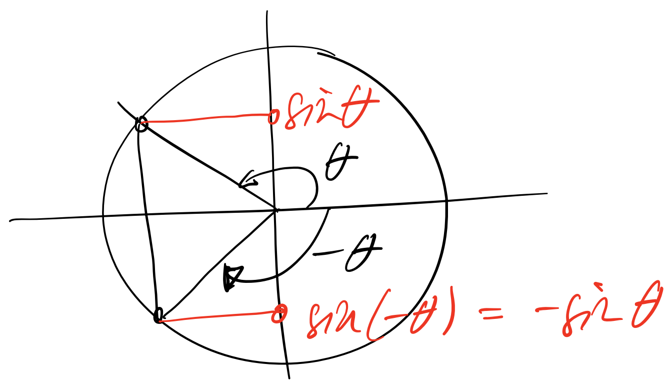
$$= \{ [1;\theta] \mid \theta \in [0;2\pi[\}$$

$$z = [1;\theta] = \cos\theta + i\sin\theta$$

$$= \cos\theta - i\sin\theta$$

$$\bar{z} = \cos\theta + i\cdot(-\sin\theta) = \cos\theta + i\cdot\sin(-\theta)$$

$$\Rightarrow \bar{z} = [1;-\theta] \in S_1$$



$$z_1 = [1;\theta_1] \quad z_2 = [1;\theta_2]$$

$$z_1 \cdot z_2 \overset{?}{\in} S_1 \qquad \frac{z_1}{z_2} \overset{?}{\in} S_1$$

$$z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \in S_1 \quad \text{car} \quad |z| = \sqrt{\frac{2}{4}+\frac{2}{4}} = 1$$

$$z+z = \sqrt{2} + \sqrt{2}i \qquad |z+z| = \sqrt{2+2} = \sqrt{4} = 2$$

$$\Rightarrow |z+z| \neq 1 \Rightarrow z+z \notin S_1$$

Contre-exemple