$$\mathbb{C} \longrightarrow \mathbb{C}$$
$$z \longmapsto \overline{z}$$

Si $z = a + bi$, $\quad \overline{z} \overset{\text{déf}}{=} a - bi$

__Propriétés:__

1) $\overline{\overline{z}} = z$

2) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

3) $\boxed{\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}}$ À démontrer pour le 07/09

$z_1 = a_1 + b_1 i$

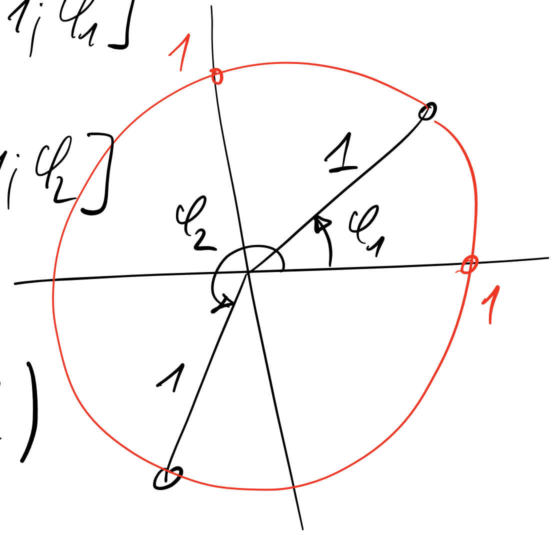
$z_2 = a_2 + b_2 i$

__Déf:__ $r \in [0; +\infty[\quad \varphi \in \mathbb{R}$

$$z = [r; \varphi] \overset{\text{déf}}{=} r(\cos\varphi + i \sin\varphi) \in \mathbb{C}$$

Fie $\quad z_1 = 1(\cos\varphi_1 + i\sin\varphi_1) \overset{not.}{=} [1; \varphi_1]$

$\quad\quad z_2 = 1(\cos\varphi_2 + i\sin\varphi_2) \overset{not.}{=} [1; \varphi_2]$



$$z_1 z_2 = (\cos\varphi_1 + i\sin\varphi_1)\cdot(\cos\varphi_2 + i\sin\varphi_2)$$

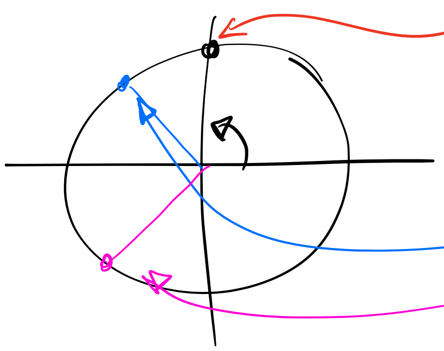
$$= (\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\varphi_2) + (\cos\varphi_1\sin\varphi_2 + \sin\varphi_1\cos\varphi_2)\cdot i$$

$$\boxed{\begin{aligned}\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \sin(\alpha+\beta) &= \cos\alpha\sin\beta + \sin\alpha\cos\beta\end{aligned}}$$

$$= \cos(\varphi_1+\varphi_2) + \sin(\varphi_1+\varphi_2)\cdot i = [1; \varphi_1+\varphi_2]$$

$$\Rightarrow z_1 z_2 = [1;\varphi_1]\cdot[1;\varphi_2] = [1; \varphi_1+\varphi_2]$$

<u>Exemplu:</u> $\quad \overset{i}{[1; \pi/2]} \cdot \overset{-\sqrt{2}/2 + \sqrt{2}/2\cdot i}{[1; 3\pi/4]} = \overset{-\sqrt{2}/2 - \sqrt{2}/2\, i}{[1; 5\pi/4]}$



Exercice : Soit $z_1 = [r_1 ; \theta_1]$ et $z_2 = [r_2 ; \theta_2]$

alors $z_1 \cdot z_2 = [r_1 ; \theta_1] \cdot [r_2 ; \theta_2] = [r_1 r_2 ; \theta_1 + \theta_2]$

$z_1 = r_1 (\cos\theta_1 + i \cdot \sin\theta_1)$

$z_2 = r_2 (\cos\theta_2 + i \sin\theta_2)$

À démontrer pour le lundi 07/09

$\boxed{1.1.4}$ $(1+i)z - iw = 2+i$

$$(1+i)z - 2 - i = i \cdot w$$

$\div i$ $\qquad$ $\div i$

$$\frac{(1+i)}{i} \cdot z - \frac{2}{i} - \frac{i}{i} = w$$

$$\left(\frac{1}{i}+1\right)z - \frac{2i}{i^2} - 1 = w$$

$$\boxed{(-i+1)\,z + 2i - 1 = w}$$

$$(2+i)\,z + (2-i)\,w = 7 - 4i$$

$$= \frac{(a+bi)(c-di)}{c^2+d^2}$$

$$\frac{a+bi}{c+di} = \frac{(a+bi)\cdot(c-di)}{(c+di)(c-di)}$$

$$\frac{1}{i} = \frac{1\cdot i}{i\cdot i} = \frac{i}{-1}$$

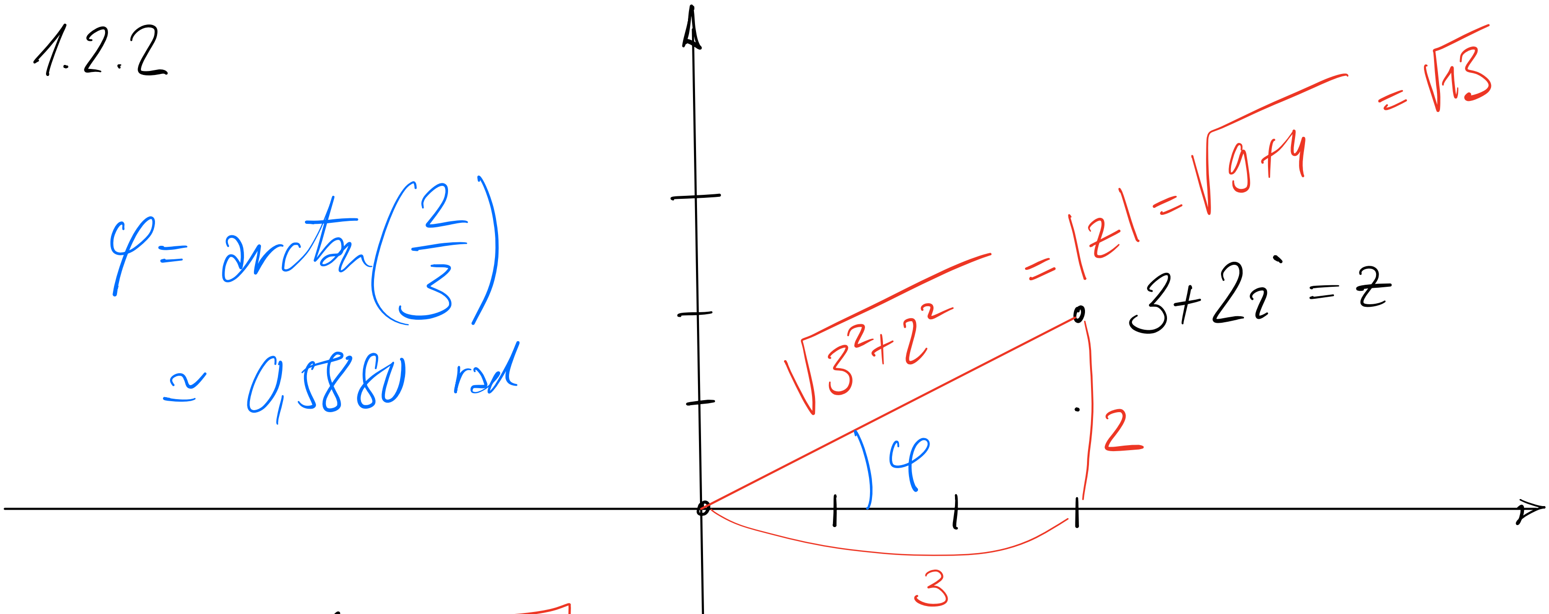
$$= -i$$

$$\frac{1}{i} = \frac{1+0i}{0+1i} \cdot \frac{0-1i}{0-1i}$$

$$= \frac{(1+0i)(0-1i)}{0^2+1^2}$$

$$= \frac{-i}{1} = -i$$

1.2.2



$$\varphi = \arctan\left(\frac{2}{3}\right)$$
$$\simeq 0{,}5880 \text{ rad}$$

$$z = \sqrt{13}\left(\boxed{\cos 0{,}5880} + i\,\boxed{\sin 0{,}5880}\right)$$
$$= \left[\sqrt{13}\,;\ 0{,}5880\right]$$

$$\varphi = \arccos\left(\frac{3}{\sqrt{13}}\right)$$

On sait que $[1;\theta]\cdot[1;\theta] = [1;\theta+\theta]$

$$= [1;2\theta]$$

$$\Rightarrow (\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$$

donne les formules pour $\cos 2\theta$ et $\sin 2\theta$

De manière analogue,

$$\underbrace{(\cos\theta + i\sin\theta)^4}_{\text{À développer avec le triangle de Pascal}} = \cos 4\theta + i\sin 4\theta$$

Racines de nombres complexes

$z = a + bi$

$r = \sqrt{a^2+b^2}$

$z = w^2$ avec $w = x + iy$

$z = (x+iy)^2 = (x^2 - y^2) + 2xyi$

$a = x^2 - y^2$ $\qquad \sqrt{a^2+b^2} = x^2+y^2 = r$

$b = 2xy$

truc : $z = w^2 \Rightarrow |z| = |w^2| = |w|^2$

$\sqrt{a^2+b^2} = x^2+y^2$

$z^4 = 1+i$ $\qquad a = 1 \qquad r = \sqrt{2}$

$(z^2)^2 = 1+i$ $\qquad b = 1$

$w^2 = 1+i$ $\qquad w = x+iy$

$$\boxed{\begin{array}{ll} 1 = x^2 - y^2 & 2xy = 1 \\ \sqrt{2} = x^2+y^2 & \end{array}}$$

À résoudre

$$1+i = \left[\sqrt{2} \, ; \, \frac{\pi}{4}\right]$$

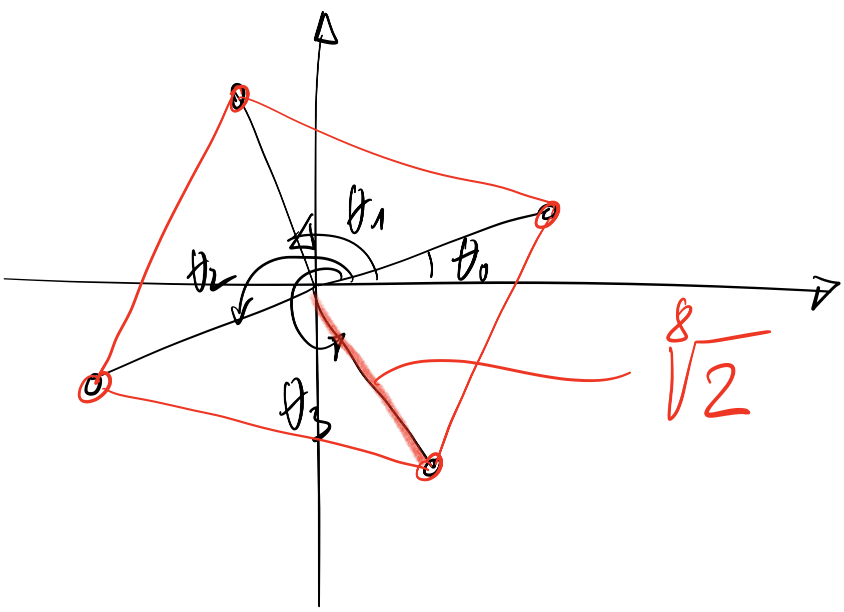
$$[r;\theta] \text{ tq. } [r;\theta]^4 = \left[\sqrt{2} \, ; \, \frac{\pi}{4}\right] = [r^4 ; 4\theta]$$

$$\sqrt{2} = r^4 \Rightarrow r = \sqrt[4]{\sqrt{2}}$$

$$\frac{\pi}{4} + k2\pi = 4\theta$$

$$\frac{\pi}{16} + k \cdot \frac{2\pi}{4} = \theta$$

$$\theta_0 = \frac{\pi}{16} \qquad \theta_1 = \frac{\pi}{16} + \frac{\pi}{2} \qquad \theta_2 = \frac{\pi}{16} + \pi \qquad \theta_3 = \frac{\pi}{16} + \frac{3\pi}{2}$$



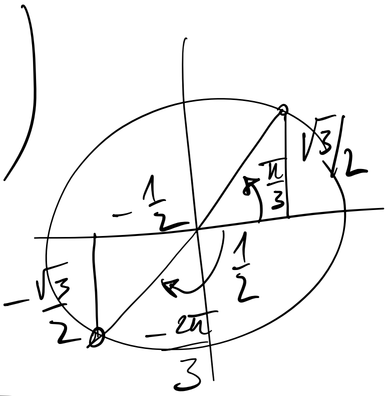
$$\left[\sqrt[8]{2} \, ; \, \frac{\pi}{16}\right]$$

$$\left[\sqrt[8]{2} \, ; \, \frac{9\pi}{16}\right]$$

$$\vdots$$

$$z = \left[\tfrac{3}{2} ; -\frac{2\pi}{3}\right]$$

$$= \frac{3}{2}\left(\cos\left(-\frac{2\pi}{3}\right) + i \sin\left(-\frac{2\pi}{3}\right)\right)$$



car $[r ; \theta] \stackrel{def}{=} r(\cos\theta + i \sin\theta)$

$$= \frac{3}{2}\left(-0{,}5 - \frac{\sqrt{3}}{2} i\right) = -\frac{3}{4} - \frac{3\sqrt{3}}{4} i$$

$$\frac{2+0i}{0+1i} \cdot \frac{0-1i}{0-1i} = \frac{-2i}{1} = -2i$$

$$\frac{2}{i} \cdot \frac{i}{i} = \frac{2i}{-1} = -2i$$

$$\frac{a+bi}{c+di} = e+fi$$

$$\frac{1}{i} \cdot \frac{i}{i} = \frac{i}{-1} = -i$$