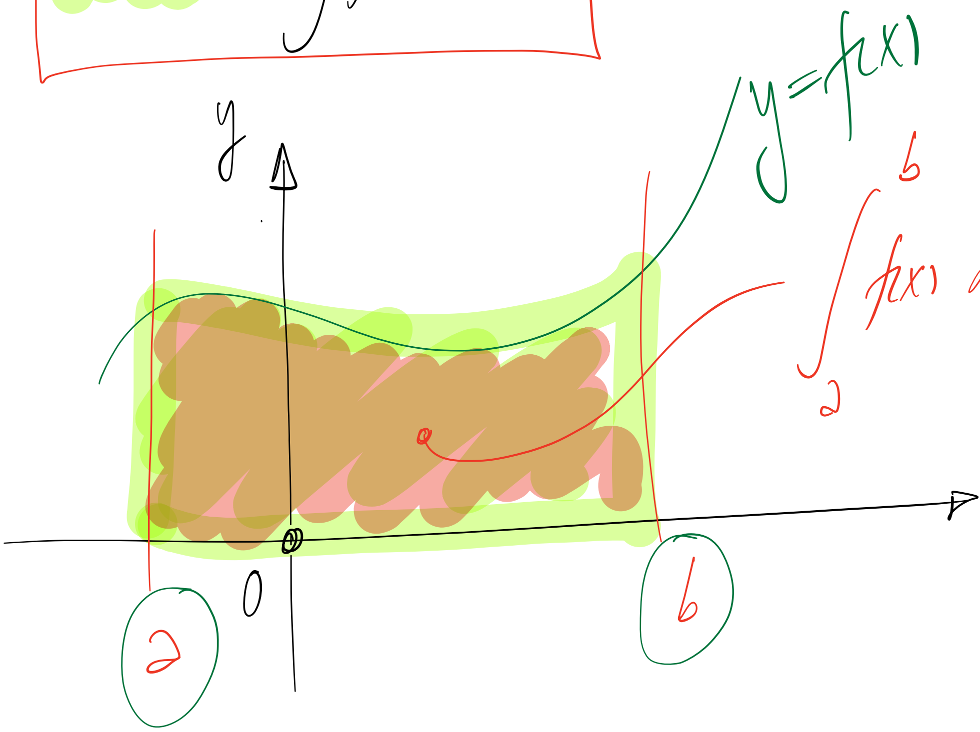
Intégrales définies

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

avec $\boxed{F'(x) = f(x)}$

$$\boxed{F(x) = \int f(x)\,dx}$$

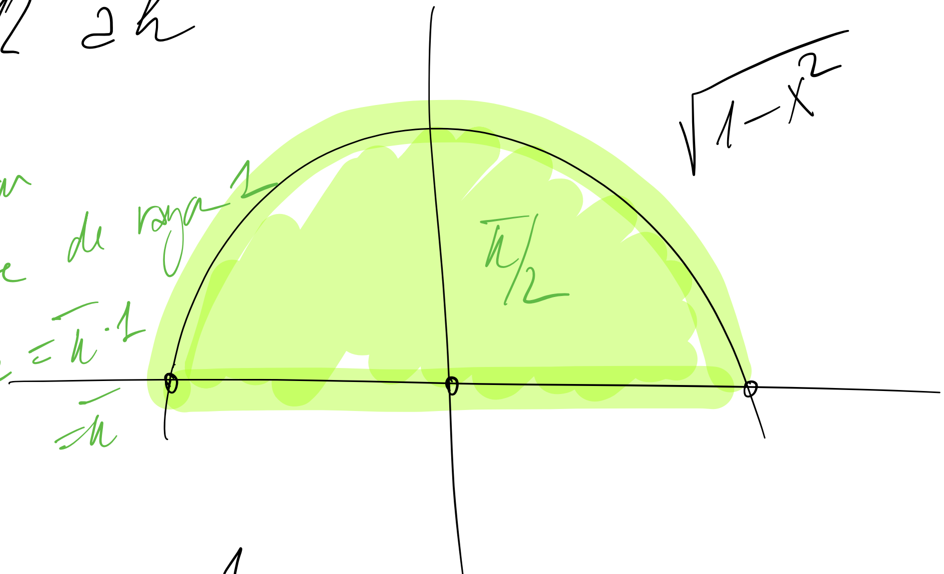


$$\int_a^b f(x)\,dx = \underbrace{\boxed{F(b) - F(a)}}_{\in \mathbb{R}}$$

2.12 ah

Aire du disque de rayon 1

$\pi r^2 = \pi \cdot 1 = \pi$



$x^2+y^2=1$

$y^2 = 1-x^2$

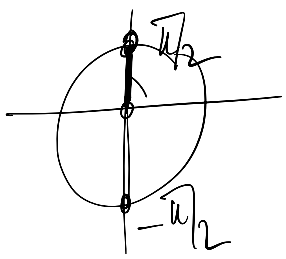
$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{1}{2}\left(x\sqrt{1-x^2} + \arcsin(x)\right)\Big|_{-1}^{1}$$

$$= \frac{1}{2}\left(\underbrace{1\sqrt{1-1^2}}_{0} + \arcsin(1)\right)$$

$$- \frac{1}{2}\left(\underbrace{-1\sqrt{1-(-1)^2}}_{0} + \arcsin(-1)\right)$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\left(-\frac{\pi}{2}\right)$$

$$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$



2.7.7 b)

$$\boxed{\sqrt{\sin x}\cdot\cos^3 x} = (\sin x)^{1/2}\cos^2 x\,\cos x$$

$$= (\sin x)^{1/2}(1-\sin^2 x)\cdot\cos x$$

$$= \sin x^{1/2}\cdot\cos x - \sin x^{5/2}\cos x$$

$$= (\sin x)^{1/2}\cdot(\sin x)' + (\sin x)^{5/2}\cdot(\sin x)'$$

$\sin x = T$

$\cos x\, dx = dT$

$$T^{1/2} - T^{5/2} \quad \int dT$$

$$\frac{2}{3}T^{\frac{3}{2}} - \frac{2}{7}T^{\frac{7}{2}}$$

$T = \sin x$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int (\sin x)^{\frac{1}{2}}\cdot\cos x\,dx = \int (\sin x)^{\frac{1}{2}}\cdot(\sin x)'\,dx$$

$$= \int \left(\frac{2}{3}(\sin x)^{\frac{3}{2}}\right)' dx$$

$$\frac{2}{3}\cdot\frac{3}{2}\cdot(\sin x)^{\frac{1}{2}}\cdot(\sin x$$

$$\int (\sin x)^{\frac{1}{2}} \cdot \underbrace{\cos x \, dx}_{dT} =$$

$$T = \sin x$$

$$\int T^{\frac{1}{2}} \, dT$$

$$dT = \cos x \, dx$$

$$\boxed{\frac{2}{3} \sin(x)^{\frac{3}{2}} - \frac{2}{7} \sin(x)^{\frac{7}{2}}} + C$$

$$\frac{2}{3} \sqrt{\sin x^{3}} - \frac{2}{7} \sqrt{\sin x^{7}}$$

$$\frac{2}{3} \sin x \sqrt{\sin x} - \frac{2}{7} \sin^{3} x \sqrt{\sin x}$$

$$\int \sin^2 x \cos^2 x \, dx$$

$$(1-\sin^2 x) \quad (1-\cos^2 x)$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{\cos(2x)+1}{2}$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\cos^2 x \cdot \sin^2 x = \frac{1-\cos^2(2x)}{4}$$