$$P(x) = x^3 - 5x^2 + 11x - 15$$

<u>But:</u> $\displaystyle\int \frac{1}{P(x)}\,dx = \int \frac{1}{x^3-5x^2+11x-15}\,dx$

① Factoriser P:

$D_{15} = \{\pm 1; \pm 3; \pm 5; \pm 15\}$

	1	-5	11	-15
3		3	-6	15
	1	-2	5	0

$$P(x) = (x-3)\underbrace{(x^2-2x+5)}_{\Delta = 4-20<0}$$

IRRÉD.

② $\displaystyle\frac{1}{P(x)} = \boxed{\frac{1}{(x-3)(x^2-2x+5)} = \frac{A}{x-3} + \frac{Bx+C}{x^2-2x+5}}$ (*)

$C, B, A \in \mathbb{R}$ $\qquad \forall x \in D_{éq.}$

$\boxed{(*) \cdot (x-3)}$

$$\frac{1}{x^2-2x+5} = A + \frac{(Bx+C)(x-3)}{x^2-2x+5}$$

$x=3$

$$\frac{1}{9-6+5} = A + 0$$

$$A = \frac{1}{8}$$

$$\boxed{\frac{1}{P(x)} = \frac{1}{8} \cdot \frac{1}{x-3} + \frac{Bx+C}{x^2-2x+5}} \quad (**)$$

$\boxed{(**) \cdot x}$

$$\frac{x}{P(x)} = \frac{1}{8} \cdot \frac{x}{x-3} + \frac{Bx^2+Cx}{x^2-2x+5}$$

$x \to \infty$

$$\frac{1}{x^2} = \frac{x}{x^3} = \frac{1}{8} \cdot \frac{\cancel{x}}{\cancel{x}} + \frac{B\cancel{x^2}}{\cancel{x^2}}$$

$$0 = \frac{1}{8} + \overset{x \to \infty}{\downarrow} B$$

$$\Rightarrow B = -\frac{1}{8}$$

$$\Rightarrow \frac{1}{x^3-5x^2+11x-15} = \frac{1}{8} \cdot \frac{1}{x-3} + \frac{-\frac{1}{8}x + C}{x^2-2x+5}$$

$\boxed{x=0}$

$$-\frac{1}{15} = -\frac{1}{24} + \frac{C}{5}$$

$$\left(\frac{1}{24} - \frac{1}{15}\right) \cdot 5 = C$$

$$\frac{15-24}{360} \cdot 5 = C$$

$$C = \frac{-9}{72} = \frac{-3}{24} = -\frac{1}{8}$$

$$\frac{1}{x^3-5x^2+11x-15} = \frac{1}{8}\cdot\frac{1}{x-3} - \frac{1}{8}\cdot\boxed{\frac{x+1}{x^2-2x+5}}$$

$$\boxed{x-1+2}$$

$$(x^2-2x+5)' = 2x-2$$

$$\int \frac{1}{P(x)}\,dx = \frac{1}{8}\ln|x-3| - \boxed{\frac{1}{8}\cdot\frac{1}{2}\int\frac{2\cdot(x-1)}{x^2-2x+5}\,dx}$$

$$\frac{f'(x)}{f(x)}$$

$$\boxed{-\frac{1}{8}\int\frac{2}{x^2-2x+5}\,dx}$$

$$= \frac{1}{8}\ln|x-3| - \frac{1}{16}\ln(x^2-2x+5)$$

$$-\frac{1}{4}\int\frac{1}{x^2-2x+5}\,dx$$

$$x^2-2x+5 = x^2-2x+1+4$$

$$= (x-1)^2+4$$

$$\boxed{x-1 = 2t} \qquad x = 2t+1 \;/\; dx = 2\,dt$$

$$t = \frac{x-1}{2}$$

$$\int\frac{1}{x^2-2x+5}\,dx = \int\frac{1}{4t^2+4}\cdot 2\,dt = \frac{1}{2}\int\frac{1}{t^2+1}\,dt$$

$$= \frac{1}{2} \arctan(t) = \frac{1}{2} \arctan\left(\frac{x-1}{2}\right)$$

$$\int \frac{1}{P(x)}\, dx = \frac{1}{8} \ln|x-3| - \frac{1}{16} \ln(x^2-2x+5)$$

$$- \frac{1}{8} \arctan\left(\frac{x-1}{2}\right) + C$$

$$\left(-\frac{1}{8}\right) \frac{x+1}{x^2-2x+5} = \left(-\frac{1}{8}\right) \frac{x-1+2}{x^2-2x+5} = \left(-\frac{1}{8}\right) \frac{x-1}{x^2-2x+5} + \left(-\frac{1}{8}\right) \frac{2}{x^2-2x+5}$$