Etude complète
d'une fonction

ED, zeros, signe

Asymptotes

Croissance ← Dérivée

Courbure ← Dérivée seconde

Graphe

$(x^n)' = n x^{n-1}$ $\quad n \in \mathbb{Q}$

$(ax+b)' = a$

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f, g dérivables

linéarité de l'opérateur

$$\boxed{\begin{aligned}(f+g)' &= f' + g' \\ (k\cdot f)' &= k\cdot f'\end{aligned}}$$

$(x^5+x^4)' = 5x^4 + 4x^3$

$(3x^7)' = 3(x^7)' = 21x^6$

$$\boxed{\begin{aligned}(f\cdot g)' &= f'g + fg' \\ \left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2}\end{aligned}}$$

$$\boxed{\left(f(g(x))\right)' = f'(g(x))\cdot g'(x)}$$

$$\left(\sqrt{x^2+1}\right)' = \left((x^2+1)^{\frac{1}{2}}\right)'$$

$$= \frac{1}{2}\cdot(x^2+1)^{-\frac{1}{2}}\cdot(x^2+1)'$$

$$\left((x+1)^5\cdot(x-3)^7\right)' = 5(x+1)^4\cdot 1\cdot(x-3)^7$$

$$+ (x+1)^5\cdot 7(x-3)^6\cdot 1$$

$$= 5(x+1)^4(x-3)^7 + 7(x+1)^5\cdot(x-3)^6$$

$(x+1)' = 1 \quad (x-3)' = 1$

$$\left(\sqrt{t}\right)' = \left(t^{\frac{1}{2}}\right)' = \frac{1}{2}t^{\frac{1}{2}-1}$$

$$= \frac{1}{2}t^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{t}}$$

$$\left(\left(x^2+x+1\right)^8\right)' = \underbrace{8\cdot\left(x^2+x+1\right)^7}\cdot\underbrace{\left(x^2+x+1\right)'}$$

$$\left(f(g(x))\right)' = f'(g(x))\cdot g'(x)$$

$$\left(t^n\right)' = n\,t^{n-1}$$

$$f(t) = t^8$$

$$t = g(x) = x^2+x+1$$

$$\boxed{h(x) = f(g(x))}$$

$$f'(t) = 8t^7$$

Deriver:

$$\left(\left(x^2-1\right)^5\right)' = 5\left(x^2-1\right)^4\cdot 2x = 10x\left(x^2-1\right)^4$$

$$\sum_{i=1}^{4}(i+1)\left(x^i\right)' = \left(\sum_{i=1}^{4}(i+1)x^i\right)' = \sum_{i=1}^{4}(i+1)\cdot i\cdot x^{i-1}$$

$$\left(\frac{x+1}{x-1}\right)' = \frac{\underbrace{(x+1)'}_{1}\overbrace{(x-1)}^{(x-1)} - \overbrace{(x+1)}^{-(x+1)}\underbrace{(x-1)'}_{1}}{(x-1)^2} = -\frac{2}{(x-1)^2}$$

$$\frac{\underbrace{1'}_{0}\cdot\sqrt{x} - 1\cdot\left(\sqrt{x}\right)'}{x} = \frac{-\frac{1}{2\sqrt{x}}}{x} = -\frac{1}{2x\sqrt{x}} = -\frac{1}{2\sqrt{x^3}} \quad \text{si } x>0$$

$$\left(\frac{1}{\sqrt{x}}\right)' = \left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2}\cdot x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2\sqrt{x^3}}$$

$$\left(2x^1+3x^2+4x^3+5x^4\right)' = 2\left(x^1\right)' + 3\left(x^2\right)' + 4\left(x^3\right)' + 5\left(x^4\right)'$$

$$= 2 + 6x + 12x^2 + 20x^3$$

$$\frac{2x-3-\sqrt{4x^2+6x}}{x} \overset{x<0}{=} 2-\frac{3}{x}+\sqrt{4+\frac{6}{x}} \xrightarrow{x\to-\infty} 4$$

$$x<0 \Rightarrow |x| = -x \Rightarrow \sqrt{x^2} = -x \Rightarrow \frac{1}{\sqrt{x^2}} = -\frac{1}{x}$$

$$= \frac{2x}{x} - \frac{3}{x} \underbrace{\boxed{-\frac{1}{x}}}_{\frac{1}{\sqrt{x^2}}} \cdot \sqrt{4x^2+6x} = \frac{2x}{x} - \frac{3}{x} + \frac{\sqrt{4x^2+6x}}{\sqrt{x^2}}$$