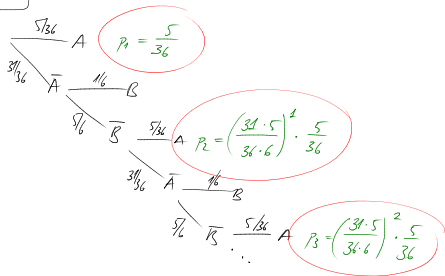


9.19



Calcul de la probabilité  
que A gagne:

$$P_n = \left(\frac{155}{216}\right)^{n-2} \cdot \frac{5}{36}$$

$$\sum_{i=1}^{\infty} P_i = P_1 + P_2 + P_3 + \dots \quad \triangle \text{ Convergence?}$$

$$\lim_{n \rightarrow \infty} \underbrace{(P_{n+1} - P_n)}_{S_n}$$

$$S_n = \left(\frac{155}{216}\right)^0 \cdot \frac{5}{36} + \left(\frac{155}{216}\right)^1 \cdot \frac{5}{36} + \dots + \left(\frac{155}{216}\right)^{n-2} \cdot \frac{5}{36}$$

$$= \frac{5}{36} (1 + 2 + 2^2 + \dots + 2^{n-2})$$

Rappel:  $(1 + x + x^2 + \dots + x^{n-2}) (1 - x) =$

$$1 + x + x^2 + \dots + x^{n-2} + x^{n-1} + x^{n-1} + x^{n-2} + \dots + x^{n-2} - x^{n-1} - x^{n-2} - \dots - x^{n-2} - x^{n-1} = 1 - x^{n-1}$$

$$\Rightarrow (1 + x + x^2 + \dots + x^{n-2}) = \frac{1 - x^{n-1}}{1 - x}$$

$$\Rightarrow S_n = \frac{5}{36} \cdot \left(\frac{1 - 2^n}{1 - 2}\right)$$

$$= \frac{5}{36} \cdot \frac{1 - \left(\frac{155}{216}\right)^n}{216 - 155}$$

$$= \frac{5}{36} \cdot \frac{216}{61} \left(1 - \left(\frac{155}{216}\right)^n\right)$$

$$\Rightarrow P(A) = \sum_{i=1}^{\infty} P_i = \frac{30}{61}$$

$$\frac{5}{36} \cdot \frac{216}{61} \cdot 1$$