a) $$\lim_{x \to 2} \frac{e^x - e^2}{x-2} = \left\langle\!\left\langle \frac{e^2 - e^2}{2-2} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle$$

Le calcul direct conduit à une indétermination.

B.-H. donne: $$\lim_{x \to 2} \frac{e^x - 0}{1 - 0} = \lim_{x \to 2} e^x$$

$$= e^2 = \lim_{x \to 2} \frac{e^x - e^2}{x-2}$$

b) $$\lim_{x \to 0} \frac{e^x - 1}{\sin(x)} = \left\langle\!\left\langle \frac{e^0 - 1}{0} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{0}{0} \right\rangle\!\right\rangle \ \underline{\text{IND}}$$

$$\lim_{x \to 0} \frac{(e^x - 1)'}{(\sin(x))'} = \lim_{x \to 0} \frac{e^x - 0}{\cos(x)} = \lim_{x \to 0} \frac{e^x}{\cos(x)}$$

$$= \left\langle\!\left\langle \frac{e^0}{\cos(0)} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{1}{1} \right\rangle\!\right\rangle = 1 \underset{\text{B.-H.}}{=} \lim_{x \to 0} \frac{e^x - 1}{\sin(x)}$$

c) $\lim\limits_{x \to 0} \dfrac{x \cdot e^x}{1-e^x} = \ll \dfrac{0 \cdot e^0}{1-e^0} \gg = \ll \dfrac{0}{0} \gg$ IND

$$\lim_{x \to 0} \frac{(x \cdot e^x)'}{(1-e^x)'} = \lim_{x \to 0} \frac{x' e^x + x(e^x)'}{0 - e^x}$$

$$= \lim_{x \to 0} \frac{e^x + xe^x}{-e^x} = \ll \frac{e^0 + 0 \cdot e^0}{-e^0} \gg$$

$$= \ll \frac{1+0}{-1} \gg = \ll \frac{1}{-1} \gg = -1$$

B.-H.

$$\Rightarrow \lim_{x \to 0} \frac{x \cdot e^x}{1-e^x} = -1$$

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d) $\lim_{x \to 0^{>}} x \cdot e^{1/x} = \langle\langle 0 \cdot e^{+\infty} \rangle\rangle = \langle\langle 0 \cdot \infty \rangle\rangle$

$\lim_{x \to 0^{>}} x \cdot e^{1/x} = \lim_{x \to 0^{>}} \frac{e^{1/x}}{1/x}$ On dérive en haut / en bas

$\lim_{x \to 0^{>}} \frac{\left(e^{1/x}\right)'}{\left(1/x\right)'} = \lim_{x \to 0^{>}} \frac{e^{1/x} \cdot \left(1/x\right)'}{\left(1/x\right)'}$

$= \lim_{x \to 0^{>}} e^{1/x} = \langle\langle e^{+\infty} \rangle\rangle = +\infty$

$\underset{B.-H.}{\Rightarrow} \lim_{x \to 0^{>}} x \cdot e^{1/x} = +\infty$