a) $$\lim_{x \to 0} \frac{\ln(x+1)}{x} = «\frac{\ln(0+1)}{0}» = «\frac{0}{0}»$$

IND.

$$\lim_{x \to 0} \frac{(\ln(x+1))'}{x'} = \lim_{x \to 0} \frac{\frac{1}{x+1}}{1}$$

$$= \lim_{x \to 0} \frac{1}{x+1} = «\frac{1}{0+1}»$$

$$= 1$$

B.-H.

$$\Rightarrow \lim_{x \to 0} \frac{\ln(x+1)}{x} = 1$$

b) $$\lim_{x \to 0} \frac{\ln(\cos(x))}{x^2} = «\frac{\ln(\overbrace{\cos(0)}^{1})}{0^2}» = «\frac{0}{0}»$$

IND.

$$, (\ln(u))' = \frac{u'}{u}$$

$$\lim_{x \to 0} \frac{(\ln(\cos(x)))'}{(x^2)'} = \lim_{x \to 0} \frac{\frac{-\sin(x)}{\cos(x)}}{2x}$$

$$= \lim_{x \to 0} \frac{-\tan(x)}{2x} = \ll \frac{-\tan(0)}{2 \cdot 0} \gg = \ll \frac{0}{0} \gg$$

IND.

$$\lim_{x \to 0} \frac{(-\tan(x))'}{(2x)'} = \lim_{x \to 0} \frac{-\frac{1}{\cos^2(x)}}{2}$$

$$= \lim_{x \to 0} -\frac{1}{2\cos^2(x)} = \ll -\frac{1}{2\cos^2(0)} \gg$$

$$= -\frac{1}{2} \quad \text{car} \quad (\underbrace{\cos(0)}_{1})^2 = 1$$

B.-H.

$$\Rightarrow \lim_{x \to 0} \frac{\ln(\cos(x))}{x^2} = -\frac{1}{2}$$

c) $$\lim_{x \to e} \frac{\ln(x) - 1}{x - e} = \ll \frac{\overbrace{\ln(e)}^{1} - 1}{e - e} \gg = \ll \frac{0}{0} \gg$$

IND.

$$\lim_{x \to e} \frac{(\ln(x) - 1)'}{(x - e)'} = \lim_{x \to e} \frac{\frac{1}{x} - 0}{1 - 0} = \lim_{x \to e} \frac{1}{x}$$

$$= \ll \frac{1}{e} \gg = \frac{1}{e}$$

B.-H.

$$\Longrightarrow \lim_{x \to e} \frac{\ln(x) - 1}{x - e} = \frac{1}{e}$$

d) $$\lim_{x \to 2} \frac{\ln(x^2-3)}{2-x} = \ll \frac{\ln(\overbrace{4-3}^{1})}{2-2} \gg = \ll \frac{0}{0} \gg$$

ind.

$$\lim_{x \to 2} \frac{\left(\ln(x^2-3)\right)'}{(2-x)'} = \lim_{x \to 2} \frac{\frac{(x^2-3)'}{x^2-3}}{(2-x)'}$$

$$= \lim_{x \to 2} \frac{\frac{2x}{x^2-3}}{0-1} = \lim_{x \to 2} -\frac{2x}{x^2-3}$$

$$= \ll -\frac{2 \cdot 2}{4-3} \gg = -4$$

B.-H.

$$\Longrightarrow \lim_{x \to 2} \frac{\ln(x^2-3)}{2-x} = -4$$

e) $$\lim_{x \to -\infty} \frac{\ln(x^2+1)}{x} = \text{«} \frac{\ln(\overbrace{(-\infty)^2+1}^{+\infty})}{-\infty} \text{»} = \text{«} \frac{+\infty}{-\infty} \text{»}$$

IND.

$$\lim_{x \to -\infty} \frac{(\ln(x^2+1))'}{(x)'} = \lim_{x \to -\infty} \frac{\frac{(x^2+1)'}{x^2+1}}{1}$$

$$= \lim_{x \to -\infty} \frac{2x}{x^2\underbrace{+1}_{\text{On peut négliger cela}}} = \lim_{x \to -\infty} \frac{2x}{x^2} = \lim_{x \to -\infty} \frac{2}{x}$$

$$= \text{«} \frac{2}{-\infty} \text{»} = 0$$

B.-H.

$$\Rightarrow \lim_{x \to -\infty} \frac{\ln(x^2+1)}{x} = 0$$

f) $$\lim_{x \to +\infty} \frac{\ln(x)+1}{1-\ln(x)} = \text{«} \frac{+\infty+1}{1-\infty} \text{»} = \text{«} \frac{+\infty}{-\infty} \text{»}$$

IND.

$$\lim_{x \to +\infty} \frac{(\ln(x)+1)'}{(1-\ln(x))'} = \lim_{x \to +\infty} \frac{\frac{1}{x}+0}{0-\frac{1}{x}} = \lim_{x \to +\infty} \frac{1/x}{-1/x}$$

$$= \lim_{x \to +\infty} \frac{1}{\cancel{x}} \cdot \frac{-\cancel{x}}{1} = \lim_{x \to +\infty} -1 = -1$$

$$\boxed{\text{Notation: } \ln^2(x) = \left(\ln(x)\right)^2}$$

$$\overset{\text{B.-H.}}{\Longrightarrow} \lim_{x \to +\infty} \frac{\ln(x)+1}{1-\ln(x)} = -1$$

$$g)\ \lim_{x \to +\infty} \frac{\ln(x^2)}{\ln^2(x)} = \ll \frac{\ln\left((+\infty)^2\right)}{\ln^2(+\infty)} \gg = \ll \frac{+\infty}{+\infty} \gg$$

$$\lim_{x \to +\infty} \frac{\left(\ln(x^2)\right)'}{\left[(\ln(x))^2\right]'} = \lim_{x \to +\infty} \frac{\frac{(x^2)'}{x^2}}{2\ln(x)\cdot\frac{1}{x}}$$

$$= \lim_{x \to +\infty} \frac{\frac{2x}{x^2}}{2\ln(x)\cdot\frac{1}{x}} = \lim_{x \to +\infty} \frac{\cancel{2}\cancel{x}}{\cancel{x^2}\cdot\cancel{2}\cdot\ln(x)} \cdot \frac{\cancel{x}}{1}$$

$$= \lim_{x \to +\infty} \frac{1}{\ln(x)} = \ll \frac{1}{\ln(+\infty)} \gg = \ll \frac{1}{+\infty} \gg = 0$$

$$\overset{\text{B.-H.}}{\Longrightarrow} \lim_{x \to +\infty} \frac{\ln(x^2)}{\ln^2(x)} = 0$$

h) $$\lim_{x \to +\infty} \frac{\ln(x)}{e^x} = \ll \frac{+\infty}{+\infty} \gg$$

INDÉTERMINÉ

$$\lim_{x \to +\infty} \frac{(\ln(x))'}{(e^x)'} = \lim_{x \to +\infty} \frac{1/x}{e^x}$$

$$= \lim_{x \to +\infty} \frac{1}{x} \cdot \frac{1}{e^x} = \lim_{x \to +\infty} \frac{1}{x \cdot e^x}$$

$$= \ll \frac{1}{+\infty \cdot +\infty} \gg = \ll \frac{1}{+\infty} \gg = 0$$

B.-H.

$$\Longrightarrow \lim_{x \to +\infty} \frac{\ln(x)}{e^x} = 0$$