a) $\cos^2 x = \cos^2\left(\frac{2x}{2}\right) = \frac{1+\cos(2x)}{2}$

$$\int \cos^2 x \, dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)\, dx$$

$$= \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\int \cos(2x)\cdot 2\, dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$$

b) $\int \sin^2 x \, dx = \int (1-\cos^2 x)\, dx$

$$= \int dx - \int \cos^2 x \, dx = x - \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

$$= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

c) $\cos(3x) = \cos x(4\cos^2 x - 3)$

$\Leftrightarrow \cos(3x) = 4\cos^3 x - 3\cos x$

$\Leftrightarrow \cos^3 x = \dfrac{\cos(3x) + 3\cos x}{4}$

$\int \cos^3 x = \frac{1}{4}\int \cos(3x)\,dx + \frac{3}{4}\int \cos x\,dx$

$= \frac{1}{12}\int \cos(3x)\cdot 3\,dx + \frac{3}{4}\sin x$

$= \dfrac{\sin 3x}{12} + \dfrac{3\sin x}{4} + C$

$= \dfrac{3\sin x - 4\sin^3 x}{12} + \dfrac{3\sin x}{4} + C = \sin x - \dfrac{\sin^3 x}{3} + C$

d) $$\int \sin^4 x \, dx = \int \sin^2 x \cdot \sin^2 x \, dx$$

$$= \int \frac{1-\cos(2x)}{2} \cdot \frac{1-\cos(2x)}{2} \, dx$$

$$= \frac{1}{4} \int \left(1-\cos(2x)\right)^2 dx$$

$$= \frac{1}{4} \int \left(1 - 2\cos(2x) + \cos^2(2x)\right) dx$$

$$= \frac{1}{4} \left[\int dx - \int \cos(2x) \cdot 2 \cdot dx + \int \cos^2(2x) \, dx \right]$$

Les primitives de la dernière lignes sont connues.