a) $(f \cdot g)' = f' \cdot g + f \cdot g'$

$f = x$, $g = -\cos x$, $f' = 1$, $g' = \sin x$

$$x \cdot (-\cos x) \qquad 1 \cdot (-\cos x) \qquad x \cdot \sin x$$

On peut écrire: $(-x \cdot \cos x)' = -\cos x + x \sin x$

$$\Rightarrow \quad -x \cos x = -\int \cos x \, dx + \int x \sin x \, dx$$

$$\Leftrightarrow \quad \int x \sin x \, dx = -x \cos x + \int \cos x \, dx$$

$$= -x \cos x + \sin x$$

Ainsi, $\displaystyle\int_0^{\pi} x \sin x \, dx = \left(-x \cos x + \sin x\right)\Big|_0^{\pi}$

$$= -\pi \cos \pi + \sin \pi - 0 \cdot \cos 0 - \sin 0$$

$$= \pi$$

b) $(f \cdot g)' = f' \cdot g + f \cdot g'$

$\sin x \cdot \sin x \qquad \cos x \cdot \sin x \qquad \sin x \cdot \cos x$

$$\Rightarrow \quad \sin^2 x = \int \sin x \cos x \, dx + \int \sin x \cos x \, dx$$

$$= 2 \int \sin x \cos x \, dx$$

$$\Leftrightarrow \quad \int \sin x \cos x \, dx = \frac{1}{2} \sin^2 x$$

$$\int_0^{\pi/2} \sin x \cos x \, dx = \frac{1}{2} \sin^2 x \Big|_0^{\pi/2} = \frac{1}{2} \sin^2 \frac{\pi}{2} - \frac{1}{2} \sin^2 0$$

$$= \frac{1}{2}$$

c) $(f \cdot g)' = f' \cdot g + f \cdot g'$

$\cos x \cdot \sin x \qquad -\sin x \cdot \sin x \qquad \cos x \cdot \cos x$

$$\cos x \sin x = \int -\sin^2 x \, dx + \int \cos^2 x \, dx$$

$$\Leftrightarrow \quad \sin x \cos x = -\int (1 - \cos^2 x) \, dx + \int \cos^2 x \, dx$$

$$= -\int 1 \, dx + 2 \int \cos^2 x \, dx$$

$$\Leftrightarrow \quad 2 \int \cos^2 x \, dx = x + \sin x \cos x$$

$$\Leftrightarrow \quad \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin x \cos x}{2}$$

$$\Rightarrow \int_0^{\pi/4} \cos^2 x \, dx = \left(\frac{x}{2} + \frac{\sin x \cos x}{2} \right) \Bigg|_0^{\pi/4}$$

$$= \frac{\pi}{8} + \frac{\sin \pi/4 \, \cos \pi/4}{2} - \left(\frac{0}{2} + \frac{\sin 0 \cdot \cos 0}{2} \right)$$

$$= \frac{\pi}{8} + \frac{\sqrt{2}/2 \cdot \sqrt{2}/2}{2} = \frac{\pi}{8} + \frac{1}{4} = \frac{2+\pi}{8}$$

d) $$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$x \cdot \frac{2}{3}(1+x)^{\frac{3}{2}} \qquad 1 \quad \underbrace{\frac{1}{\frac{1}{2}+1} \cdot (1+x)^{\frac{1}{2}+1}} \qquad x \quad \cdot (1+x)^{\frac{1}{2}}$$

$$\Rightarrow \frac{2x}{3}(1+x)^{\frac{3}{2}} = \int \frac{2}{3}(1+x)^{\frac{3}{2}} dx + \int x (1+x)^{\frac{1}{2}} dx$$

$$\Leftrightarrow \int x\sqrt{1+x}\,dx = \frac{2x}{3}(1+x)^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{1}{\frac{3}{2}+1}(1+x)^{\frac{3}{2}+1}$$

$$= \frac{2}{3}\left[x(1+x)^{\frac{3}{2}} - \frac{2}{5}(1+x)^{\frac{5}{2}}\right]$$

$$\Rightarrow \int_0^3 x\sqrt{1+x}\,dx =$$

$$\frac{2}{3}\left[3\cdot 4^{\frac{3}{2}} - \frac{2}{5}\cdot 4^{\frac{5}{2}}\right] - \frac{2}{3}\left[0 - \frac{2}{5}\cdot 1^{\frac{5}{2}}\right] =$$

$$\frac{2}{3}\left[3\cdot 8 - \frac{2}{5}\cdot 32\right] + \frac{4}{15} =$$

$$\frac{48}{3} - \frac{128}{15} + \frac{4}{15} = \frac{240-124}{15} = \frac{116}{15}$$