$$A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

$$A' = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \cdot \left(\overset{u_1,\ \lambda_1 = 2}{\boxed{\begin{matrix} -1 \\ 1 \end{matrix}}} \ \overset{u_2,\ \lambda_2 = 3}{\boxed{\begin{matrix} -1 \\ 2 \end{matrix}}} \right)$$

$$= \underbrace{\begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}}_{P^{-1}} \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \underbrace{\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}}_{P}$$

$$\underbrace{A = P \overbrace{\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}}^{A'} P^{-1}}_{n=1}$$

$\boxed{n \Rightarrow n+1}$

$$A^{n+1} = A^n \cdot A = P \cdot (A')^n \cdot P^{-1} \cdot A$$

$$= P\,(A')^n \cdot \underbrace{P^{-1} \cdot P}_{I_2} \cdot A' \cdot P^{-1}$$

$$= P\,(A')^n \cdot A' \cdot P^{-1}$$

$$= P \cdot (A')^{n+1} \cdot P^{-1}$$

(QFD)

$$A^n = \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2^n & 0 \\ 0 & 3^n \end{pmatrix} \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2^{n+1} - 3^n & 2^n - 3^n \\ -2^{n+1} + 2 \cdot 3^n & -2^n + 2 \cdot 3^n \end{pmatrix}$$