a) $(-1)^2 + (2)^2 = 1+4 = 5$ ✓

$\Rightarrow T \in \mathcal{H}$

$\overrightarrow{CT} = (-1-0;\ 2-0) = (-1;2)$

$t:\ -x+2y+k=0$ passant par T

$\Rightarrow\ 1+4+k=0 \ / \ k=-5$

$\Rightarrow\ t:\ -x+2y-5=0 \iff x-2y+5=0$

b) $(-5+2)^2 + (7-3)^2 = (-3)^2 + 4^2 = 9+16$

$\Rightarrow T \in \mathcal{H}$ $= 25$ ✓

$t:\ (-5+2)(x+2) + (7-3)(y-3) = 25$

$\Leftrightarrow \quad -3x - 6 + 4y - 12 = 25$

$\Leftrightarrow \quad -3x + 4y - 43 = 0$ est l'équation de t.

$\Leftrightarrow \quad 3x - 4y + 43 = 0$

c) $\mathcal{K}: x^2 + y^2 - 3x + 7y = 0$

$\Leftrightarrow \quad x^2 - 2x \cdot \frac{3}{2} + \left(\frac{3}{2}\right)^2 + y^2 + 2 \cdot y \cdot \frac{7}{2} + \left(\frac{7}{2}\right)^2$

$= \frac{9}{4} + \frac{49}{4} = \frac{58}{4}$

$\Leftrightarrow \boxed{\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{7}{2}\right)^2 = \frac{58}{4}}$

$C\left(\frac{3}{2}; -\frac{7}{2}\right) \quad r = \frac{1}{2}\sqrt{58}$

$$\left(-\frac{3}{2}\right)^2 + \left(\frac{7}{2}\right)^2 = \frac{9}{4} + \frac{49}{4} = \frac{58}{4} = \frac{29}{2} \checkmark$$

$\Rightarrow T \in \mathcal{C}$

$\vec{CT} = \left(-\frac{3}{2}; \frac{7}{2}\right)$

$\Rightarrow t: -\frac{3}{2}x + \frac{7}{2}y + k = 0$ passant par $(0;0)$

$\Rightarrow k = 0 \Rightarrow t: -3x + 7y = 0$

$\Leftrightarrow \boxed{t: 3x - 7y = 0}$

d) $\mathcal{C}: (x-1)^2 + (y+3)^2 = 29$

$t: (-1-1)(x-1) + (2+3)(y+3) = 29$

$\Leftrightarrow -2x + 2 + 5y + 15 = 29$

$\Leftrightarrow \quad 2x - 5y + 12 = 0$

L'équation de t est donc $2x - 5y + 12 = 0$

e) $\mathcal{H}: \quad x^2 - \frac{1}{2}x + y^2 - 2y = 6$

$\Leftrightarrow \left(x - \frac{1}{4}\right)^2 + (y-1)^2 = 6 + \frac{1}{16} + 1$

$\Leftrightarrow \left(x - \frac{1}{4}\right)^2 + (y-1)^2 = \frac{113}{16}$ $\qquad C\left(\frac{1}{4}; 1\right)$

$2^2 - \frac{1}{2} \cdot 2 + 3^2 - 2 \cdot 3 = 6$

$\Leftrightarrow \quad 4 - 1 + 9 - 6 = 6$

$\Leftrightarrow \quad 3 + 3 = 6 \; \checkmark \quad \Rightarrow T \in \mathcal{H}$

$\overrightarrow{CT} = \left(2 - \frac{1}{4}; 3 - 1\right) = \left(\frac{7}{4}; 2\right)$

$\Rightarrow t: \; \frac{7}{4}x + 2y + k = 0$ passant par T

$$\Rightarrow \frac{7}{4} \cdot 2 + 2 \cdot 3 + k = 0$$

$$\Rightarrow k = -6 - \frac{7}{2} = -\frac{19}{2}$$

$$\Rightarrow t: \frac{7}{4}x + 2y - \frac{19}{2} = 0$$

$$\Leftrightarrow t: 7x + 8y - 38 = 0$$

f) $k: x^2 - \frac{2}{3}x + y^2 = \frac{11}{3}$

$$\left(x - \frac{1}{3}\right)^2 + y^2 = \frac{11}{3} + \frac{1}{9} = \frac{34}{9}$$

$$t: \left(2 - \frac{1}{3}\right)\left(x - \frac{1}{3}\right) + y = \frac{34}{9}$$

$$\frac{5}{3}x - \frac{5}{9} + y = \frac{34}{9} \Rightarrow 15x + 9y = 39$$

$$\Leftrightarrow \boxed{t: 5x + 3y = 13}$$